Citation: Subramaniam, K. & Padalkar, S. (2009). Visualisation and reasoning in explaining the phases of the moon. International Journal of Science Education, Vol 31(3), Special Issue on "Visual and Spatial Modes in Science Learning". pp. 395-417.

Visualisation and Reasoning in Explaining the Phases of the Moon

K. Subramaniam and Shamin Padalkar
Homi Bhabha Centre for Science Education, TIFR

Abstract

In this study we examine how subjects set up, transform and reason with models that they establish on the basis of known facts as they seek to explain a familiar everyday phenomenon – the phases of the moon. An interview schedule was designed to elicit subjects' reasoning, and in the case where explanations were mistaken, to induce a change in explanation. Detailed interviews of eight participants were videotaped and their reasoning analysed to highlight the difficulties encountered, the interaction between physical and geometrical aspects, simplification and idealisation processes, interplay between facts, concepts and visualisation, and the use of external visualisations through gestures and diagrams. We suggest that visualisation is an important process in science learning, and point to the importance of developing among students the ability to work with diagrams.

Introduction

Researchers have recently called attention to the potential benefits of harnessing imagery and visuospatial reasoning in science learning (Gilbert, 2005). This body of work finds an echo in studies in the history and philosophy of science that view Model Based Reasoning (MBR) as a core process underlying the growth of scientific knowledge (Nersessian, 1999). For Nersessian, visual modelling is one of the forms of MBR, together with analogical modelling and thought experiment. Gilbert (2005) associates MBR more closely with visualisation and characterises models as simplified descriptions of complex phenomena, chosen to aid the formation of visualisations. He also argues for a central role for visualisation in science learning, taking this to be a corollary of the fact that it plays a central role in the conduct of science.

Recent studies have investigated the role of visualisation and MBR in different domains of science learning, as for example, in chemistry, where explanations are largely based on entities and processes in the sub-micro world, or in geology, where distances and scales are too large to be grasped by direct perception. The domain of astronomy is an interesting example beyond even geological scales, and also inaccessible to direct perception. The understanding of elementary astronomical phenomena arguably forms an essential component of scientific literacy, given its roots in culture and everyday experience. Vosniadou and her colleagues (see Vosniadou & Brewer, 1992) have studied the alternative mental models that children form in the domain of astronomy, and have sought to explain the prevalence, sometimes across different cultures, of such mental models. When children are exposed to the scientific model of the earth, and such facts as that it is suspended in space and is spherical in shape, they form synthetic models.

These studies treat mental models as cognitive structures that result from learning or instruction. Such models provide a diagnostic tool to measure the success of instruction, and identifying and correcting the alternative models would be one of the goals of science education. However, students may have the correct model and still harbour important misconceptions. As children grow and are exposed to information from different sources, they absorb the common knowledge present in the culture, and the facts pertaining to a phenomenon may be corrected over time. But such 'knowledge' may remain unexamined and only vaguely understood. The processes that constitute scientific thinking complement and complete the understanding gained on the basis of information and facts.

Nersessian (2002) contrasts the interpretation of mental models as structures in long term memory and as structures in working memory. She emphasises the process aspect of models, or 'modelling', which is an important component of scientific thinking. Knowing what models students have or form when probed illuminates one facet of students' knowledge. Equally importantly, we also need to know what they do with the models. We need to understand how students generate, modify and elaborate models, and how they use them in making inferences. In the domain of elementary astronomy, knowing the correct facts often does not amount to a complete understanding. Many familiar daily phenomena such as the day-night cycle, the seasons, the appearance and position of the sun and the moon, can be explained on the basis of well known elementary astronomical facts. These explanations are nevertheless surprising and cannot be arrived at spontaneously. They often require one to construct and reason with models by imagining, representing and transforming them in order to arrive at conclusions. In this study we examine the reasoning processes that underlie the explanation of the phases of the moon, the visuospatial dimensions of such reasoning and the conceptual change that results.

Elementary astronomical facts are sufficient to completely explain the puzzling changes in the moon's shape. The moon does not emit any light of its own, and at all times half the moon's surface is lit by the sun, which is at a distance of nearly 40,000 moon diameters. The phases of the moon are determined by how much of its lit part is seen from the earth as the moon moves around it. The first recorded scientific and correct explanation of the moon's phases is by the philosopher Anaxagoras who lived in Athens in the 5th Century BC (Heath, 1931). He argued that since the bright face of the moon is always turned towards the sun, the moon must shine by the light of the sun. This brilliant piece of reasoning is cited by Aristotle (Posterior analytics, Book 1, chapter 33) as an example of 'quick wit' or the instantaneous perception of the middle term of a syllogism, a form of reasoning that we now call 'abduction' (Magnani, 2001).

Given the emphasis laid on elementary astronomy as a part of scientific literacy, it comes as a surprise and as a matter of concern, that there exists a widespread misconception about the mechanism underlying the phases of the moon, even among those who have had a college education. Earlier studies have found that many students and adults (varying from 14% to 70% of the sample in different studies, Trundle et al., 2002) think that the phases of the moon occur due to the shadow of the earth falling on the moon.

(Hereafter we shall call this mechanism proposed to explain the moon's phases the 'eclipse mechanism' and the explanation based on the eclipse mechanism the 'shadow explanation'.) The shadow explanation is correct for the lunar eclipse, but not for the lunar phases. Other incorrect explanations such as 'cloud covers the moon', 'planet or the sun casts a shadow on the moon' have also been found among school students. In one study, grade 3 students who underwent instruction on this topic developed the following alternative mechanism: the phase of the moon is related to one's position on the spherical earth (Stahly et. al., 1999). Since the distance to the moon is roughly 30 times the diameter of the earth, an observer's location on the earth does not significantly affect the shape of the phase of the moon. Of course, at a given point of time, the position of the observer on a particular terrestrial location would determine whether the moon is visible at all. For example, if the moon is overhead for a given location, it cannot be seen by an observer standing at the antipode.

The cognitive basis of wrong explanations of the moon's phases is different from that of holding alternative mental models on which previous studies in the domain of astronomy have focused. The present study tries to understand, given that the basic facts pertaining to the Earth-Sun-Moon (ESM) system and the basic 'mental model' are correct, what are the explanations offered by participants and how do participants change these explanations when inconsistencies are pointed out. The mental model relevant to the problem includes the spatial and motion properties of, in this case, the ESM system. The nature of reasoning required to solve the problem of the moon's phases involves relating the spatial properties of bodies such as shape, size, position, motion, etc. to arrive at inferences. Such reasoning may be described as model-based visuospatial reasoning and would involve reasoning with external and internal images or other visual representations in addition to proposition based reasoning. It may include processes such as mental rotation, perspective taking or other transformations and may involve working with analogies. In the context of the problem identified for study, the following questions were of interest to us: Do people use visuospatial reasoning to solve this problem? How does visuospatial reasoning interact with verbal proposition based reasoning? What mental tools and resources, and especially, what external visual representations do people draw upon? What kind of strategies, devices or skills are helpful for such reasoning?

Design of the study

The study consisted of initial informal interviews, a pilot study based on a written questionnaire and interview, and the main study, also based on a written questionnaire and interview. There were eight participants in the main study, which forms the focus of this report.

In the initial informal interviews, people from different backgrounds were asked about the reason for the change in the moon's appearance and most of them offered the eclipse mechanism as the explanation for the moon's phases. Based on these interviews a written questionnaire was prepared to elicit participants' explanation of the lunar phases, which was administered to six pilot participants and was refined based on their responses. Next, a detailed schedule for an interview aimed at inducing a change in explanation using hints was drawn up, piloted with two participants and thus further refined.

It was clear from the informal interviews and the pilot study that the basic model of the earth orbiting the sun and the moon orbiting the earth was correct for nearly all participants. So the main study focused on how participants reasoned on the basis of the correct orbital model. An examination of the responses to the informal interviews, and to the pilot questionnaire and interviews, combined with an analysis of the problem, led to the identification of two aspects: (i) the mechanism that related the change in the moon's position to its changing appearance from the earth, and (ii) the precise shape of the moon's phases. These two aspects of the problem seemed to require different steps of reasoning, and a successful solution of the first part did not necessarily lead to clarity on the second aspect. Both these aspects of the problem were probed in the main study.

Sample: There were eight participants in the main study. Selection was based on their prior degree (architecture or physics), gender and convenience. Two female and two male participants were students of a 'Master of Design' programme, with a college degree in architecture and a 'visual communication' specialisation in their current course. The remaining two female and two male participants had a Master's degree in Physics and were working on projects related to physics education. The architecture and physics groups were expected to have contrasting and complementary capabilities relevant to solving the task – experience with making and using visual representations, and knowledge of physics respectively. The participants are referred to by pseudonyms beginning with the first eight letters of the alphabet. Names beginning with A through D designate architects, while E through H designate physicists. Occasionally they will be referred to by the first letter abbreviation. A, B, E and F, whose names end in the vowel 'a', designate female participants. The names do not indicate the religion of the participants.

Method: In the main study, participants filled up a main written questionnaire on the explanation of the lunar phases. A data sheet, included with the main questionnaire, contained all pertinent data concerning the Earth-Sun-Moon system with regard to the sizes, distances, ratios of sizes and distances, time-periods and the angle of inclination of moon's orbit to the ecliptic. The main questionnaire was followed by two short questionnaires called Hint Sheet 1 and Hint Sheet 2. Their purpose was to see whether participants modified their incorrect explanations on the basis of a minimal set of hints. Questions in the main questionnaire and the hint sheets are given in Table 1.

Main Questionnaire: (i) draw the phases of the moon in sequence (ii) explain in words the reason for the change in the shape (iii) draw a diagram to show how the shape changes (iv) draw an accurate diagram of the moon in these five phases: full, half, less than half, more than half and new moon.
Hint Sheet 1: (i) Some people think that the phases of the moon occur because the earth's shadow falls on the moon. Do you think this is correct? Give arguments to support your answer. (ii) Do we ever see a half moon in the sky? Could this shape be caused by the earth's shadow? Give reasons.
Hint Sheet 2: These questions may help you understand better how the moon's phases occur. (i) How much of a spherical ball in a uniformly lit room is visible when we look at it? (ii) What is the shape of the boundary of the visible part? (iii) How much of a ball will be lit by a single distant light? and (iv) What is the shape of the boundary of the lit part? Questions (v) to (vi) were about the shape you see when you look at a bangle from

different angles.

Table 1: Questions in the main questionnaire and the hint sheets

The questionnaires were followed by interviews with each participant, where they were probed in detail about their proposed explanation of the lunar phases. Two interviewers were present for all interviews with one interviewer taking the leading role and the other supporting. The task of explaining the phases was restricted to the two sub-tasks of (i) explaining how the various phases occur and (ii) their precise shapes, so that it was feasible to explore it within the period of a single interview. These sub-tasks correspond to the two aspects of the problem identified in the initial analysis. The orientation of the phase in the sky was ignored for simplicity and hence the distinction between the shapes obtained during the waxing and the waning phases was not addressed.

Structure of the Interview

Interview segment 1: The first set of questions were aimed at understanding the participants' initial model and mechanism as found in their written response and accompanying diagram ('Which view have you drawn? ' 'Where is the sun', if sun is not in the picture, etc.). They were asked if they wished to modify their written responses. The next step was to challenge their proposed explanation if it was wrong. Those who had given the eclipse mechanism were asked: When does the lunar eclipse occur? Why does it happen? How long does it take?

Interview segment 2: This segment was aimed at leading the participants to a correct and detailed explanation of the mechanism underlying the moon's phases. To guide their thinking a sequence of anchoring situations were prepared in advance and a subset of them was given to individual participants, depending on their responses. These situations, which are analogous to the ESM system but involve familiar objects, were presented in purely verbal form and no props and diagrams were used (Table 2). For each situation, participants were invited to think of how the ball (for 1a, 1a', 1b, 1b') or person (for 2a and 2b) would look when viewed from different positions. The similarity and equivalence of the partial views obtained in different anchor situations were probed from time to time during the interview.

Anchor situation 1a: *Imagine that you are standing in a dark park at the centre of rotating platform. At the edge of the platform there is a white ball mounted on a stick which is lit by distant light. As the platform rotates the ball moves around you slowly. Think of how the ball would look in different positions. Can you draw a diagram to show how they will look?*
Anchor situation 1b: *As in 1a, but ball is stationary and the observer moves around the ball viewing it from different angles.*
Anchor situation 2a: *Anthropomorphic version of 1a, with the ball replaced by a friend always facing a distant light.*
Anchor situation 2b: *Anthropomorphic version of 1b, with the ball replaced by a friend always facing a distant light.*

Anchor situations 1a' and 1b': *The ball in 1a and 1b is replaced by a black and white painted ball which is fully illuminated.*

Table 2: Anchor situations described to the participants during the interview

Interview segment 3: In this segment participants were asked to determine the correct shape of moon in each of the phases. To help them arrive at the shape, they were asked about the nature and appearance of the boundary line between the lit and dark halves of the moon. Some participants were also shown a concrete model – a table tennis ball painted half black and half white, which could be turned around in the hand and viewed from different angles. Finally, to confirm their understanding, the participants were asked: “how will the earth look from the moon on the day of the full moon/ new moon/ half moon?"

In the written questionnaire as well as interview, participants were required to themselves produce the internal and external visualisations that were necessary. All hints and helpful suggestions were restricted to verbal descriptions, the only exception being a black and white painted ball shown to some participants towards the end of the interview. Complete transcripts of the interviews were prepared including information about the gestures used by interviewers and participants. The video recordings and the transcripts were reviewed repeatedly by both authors, and remarks were inserted at various points of interest. On key aspects, the relevant video and transcript segments from different participants were reviewed sequentially so that a comparative view of all the interviews on that particular aspect emerged. Time codes, entered in the transcripts approximately every minute or at the beginning of a spoken segment, have been rounded here to the nearest minute.

Results

Responses to the Written Questionnaire: The most frequent explanation of the lunar phases given by the participants involved the eclipse mechanism. Four architects and one physicist clearly and explicitly described the eclipse mechanism as the cause of the phases of the moon. Another physicist (Esha) wrote the phrase 'shadow of the earth' in her verbal explanation, suggesting that she had the eclipse mechanism in mind. This was confirmed in the interview later, when she proposed the eclipse mechanism. Gauhar, also a physicist, wrote very little and his explanation was not clear. It appeared however that he did not think of the eclipse mechanism as causing the lunar phases. Only one physics graduate, Farha, presented the correct mechanism clearly in her verbal explanation. Her diagram was consistent with the correct explanation.

Participants' diagrams were in general consistent with their verbal explanations (Question iii in the main questionnaire, Table 1). As may be expected, the architects' diagrams were clearer and more detailed. Three of them presented both `top' and `side' views of the orbital configuration, in contrast to the other five participants who presented only one view (Table 3, Row 1). All the architects drew more than one position of the moon while only two physicists did so. The architects mixed text and drawing by including captions and descriptions (e.g. Figure 1) to a greater extent than the physicists. Among all participants' diagrams, only Farha's presented an acceptable model with the correct explanation (Figure 2).

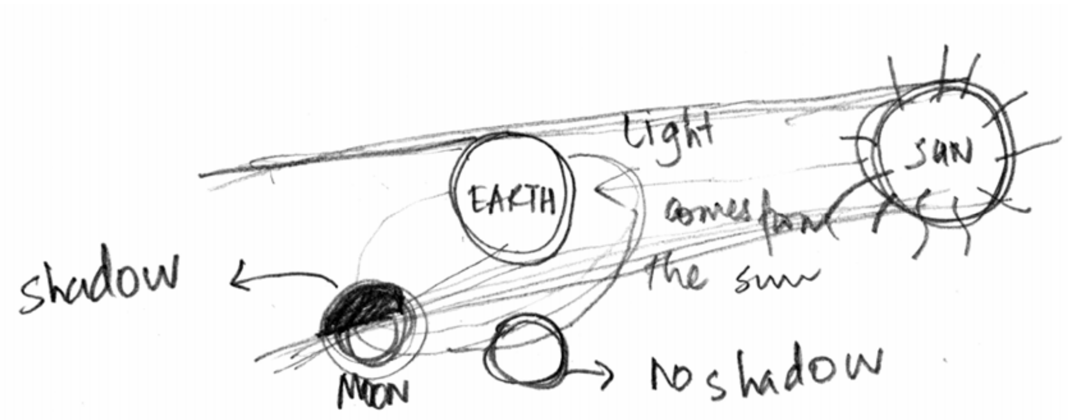


Figure 1: The shadow explanation (Asha)

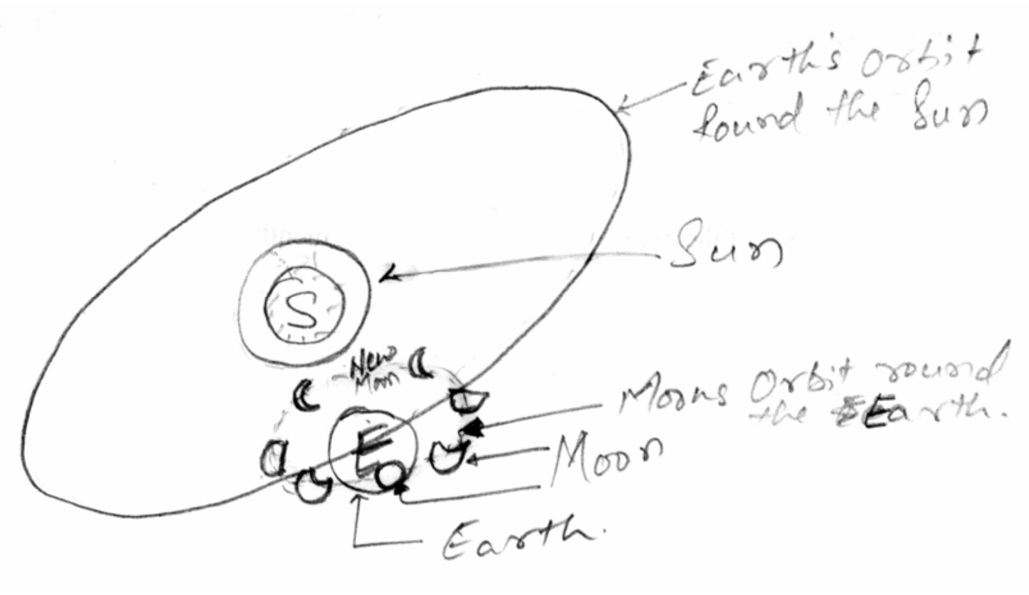


Figure 2: Farha's drawing showed the correct mechanism but wrong gibbous shape

Chander, an architect, had in addition to the eclipse mechanism, drawn a separate diagram which included the information that the moon's orbit was inclined to the ecliptic at 5 degrees and presented the correct explanation for the full moon phase. This was on a blank page of the questionnaire, not in response to any question. It emerged from the interview that Chander considered this as an alternative explanation.

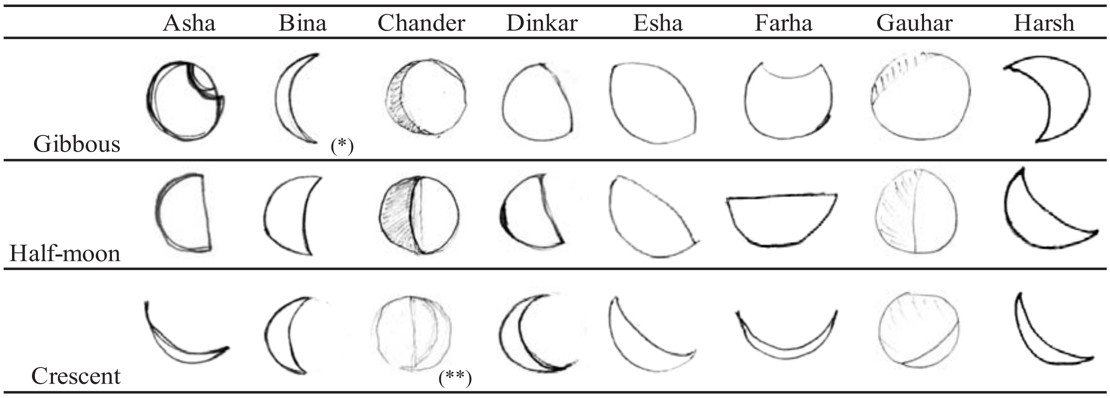
	Asha	Bina	Chander	Dinkar	Esha	Farha	Gauhar	Harsh
View (T: Top, S: Side, I: Inclined)	T	S+T	T+S	T+S	S	I	S	I
Number of Moon positions shown	2	2	Several	Several	1	Several	1	Several

Relation between Moon's Orbital Plane and Ecliptic	Unclear	Coincident	Coinci-dent/ Inclined	Coinci-dent	Inclined	Unclear	Translated*	Unclear

Gauhar had raised the moon's orbital plane well above the ecliptic, probably to avoid the shadow of the earth falling on the moon

Table 3: Summary of participants' diagrams explaining the lunar phases in the main questionnaire

Participants' drawings of the shape of the lunar phases were not accurate and also not consistent with the proposed mechanism. Table 4 shows their drawings of the phases in response to Question iv in the main questionnaire. The drawings for Question (i) were similar except in the two instances indicated in the Table. Two architects and two physicists drew an incorrect eclipse-like diagram for the gibbous phase, a shape described as 'false gibbous' by Trundle et al. (2002). However, this was not necessarily consistent with their written explanation for the phases. Farha described the correct mechanism but drew an eclipse-like diagram for the gibbous phase, while Chander, Dinkar and Esha drew the correct gibbous shape but proposed the shadow explanation. Gauhar, whose explanation was unclear and could not be categorised as correct, drew a somewhat ambiguous diagram for the gibbous phase (Table 4). It emerged from the interviews that even those who drew correct diagrams did not know that the curve defining the shape of each phase was a semi-ellipse. The participants' (in particular the physicists') difficulty in visualising and drawing the exact shapes of the phases, were further explicated during the interview.



* Bina appeared to have mistakenly drawn a crescent. Her diagram for Q1 showed an eclipse-like gibbous phase.

** For the crescent phase in Q1 Chander drew the complement of an eclipse-like gibbous phase.

Table 4: Participants' diagrams of the different phases

Hint sheet 1 (Table 1), was given to the five participants who clearly offered the eclipse mechanism. The purpose of these questions, which were phrased in a neutral manner, was to call attention to the eclipse mechanism and to give subjects a second chance to think about it. In their responses to Hint Sheet 1, four participants did not change their explanations of the lunar phases and reaffirmed the eclipse mechanism as causing the phases. One (Bina) changed her explanation at this point. In the main questionnaire she had presented the eclipse mechanism, while in response to this question, she wrote:

I think I am one of these people [who think the phases occur because of the earth's shadow]. *It is because some phase* [part?] *of the moon is not exposed to the sun because it is revolving around the earth it is not completely exposed to the sun all the time. Hence what we see from the earth is the phase* [part?] *of the moon which is illuminated by the sun. It is probably since the earth is revolving around the sun and around its own axis that we see the moon in this fashion. I guess it is not the shadow of the earth. I stand corrected.*

Hint Sheet 2 containing seven questions about the appearance of a spherical ball and a bangle under various conditions (see Table 1) was given to all participants, who wrote correct responses to most of the questions. Some responses were ambiguous, but overall, the situations appeared familiar and simple enough to visualise. The purpose of these questions was to trigger possible rethinking about the phases of the moon – a change which could be determined at the time of the interview. In fact, two respondents, Asha and Dinkar, reported in the interview that the questions in Hint Sheet 2 had made them think about the problem and led them to change their explanation. Another, Esha, recalled a question from this hint sheet as she began to change her explanation during the interview from the incorrect shadow explanation. During Chander's interview, after he became convinced of the correct explanation, he referred to the bangle questions, as he began to reason about the shapes of the various phases.

Interview Responses: Table 5 summarises the changes in the participants' explanation in the interviews. In general, the interviews with the architects (mean = 53 min) were significantly shorter than the interviews with the physicists (mean = 92 min).

	Asha	Bina	Chander	Dinkar	Esha	Farha	Gauhar	Harsh
Interview time (min.)	75	60	52	24	94	79	110	86
Dominant mechanism in questionnaire	Eclipse	Eclipse changed to correct	Eclipse, (correct alternative)	Correct	Eclipse	Eclipse	Unclear, probably correct	Eclipse
Dominant mechanism at the start of interview	Correct	Correct	Eclipse	Correct	Eclipse	Correct	Probably correct	Eclipse
Reason for change	Hint Sheet 2	Hint Sheet 1	anchor situation 1a	Hint Sheet 2	Eclipse duration anomaly	-	-	Eclipse duration anomaly

Table 5: Summary of the changes in explanation for all the participants

Interview segment 1: As seen in Table 5, Chander, Esha and Harsh reaffirmed the eclipse mechanism as causing the lunar phases at the beginning of the interview. They were then asked about the lunar eclipse: how did it occur, how long did it last, was it different from the phases? For Esha and Harsh, this led to a rethinking and to exploring alternative explanations. When Esha was asked about the durations of the eclipse and the phase cycle of the moon, she was silent for a while and when the interviewer asked her to `share her thoughts', she said, '[I am thinking if] *same mechanism* [i.e. shadow of earth] *can explain both the things* [i.e., eclipse and phases] *or there can be some other mechanism*'. Similarly, when Harsh was asked about the duration of the eclipse and of the phase cycle, he began to propose alternative explanations: '*... see suppose I am watching moon and here ... lit by ... here* (holding up one hand for the moon and the other for the sun), *sometimes I don't get the full moon, so I am not able to see the full part where light is falling. Is it?* (turns to interviewer for confirmation)'. In the case of Chander, discussion about the lunar eclipse did not lead him to reject the shadow explanation altogether. Chander appeared to simultaneously consider both the eclipse mechanism and the correct mechanism as possible explanations for the phases. Only after a discussion of Anchor Situation 1a, he was convinced that the correct explanation was the "better" one.

Table 5 also shows that at the beginning of the interview five out of the eight participants had identified the correct mechanism causing the phases. This did not mean that they could clearly explain all the phases on the basis of the correct mechanism. Many of them became confused while trying to explain all the phases and some fell back on incorrect explanations or on alternative conceptions. Farha had explained the correct mechanism quite clearly in her diagram in the written questionnaire. However, while explaining the various phases during the interview, she became unsure and began to propose the shadow explanation. At this point, she was asked about eclipses, and she quickly gave up the shadow explanation and returned to working with the correct mechanism.

The first segment of the interview pointed out to us how difficult it was to explain the phases in detail despite knowing the correct mechanism. We observed participants attempting to fit various pieces of knowledge into the explanation, and becoming confused often. Some of them attempted to make simplifications, while others attempted to take into account all the motions of the earth and the moon. Some tried to recall their observations of the moon and attempted to reconcile what they recalled with the explanation.

Interview Segment 2: The focus of this segment was the detailed explanation of particular phases, which involved specifying the relative positions of the earth, moon and sun for each phase, and accounting for how much of the moon will be visible from the earth using the given configuration of positions. This clearly involved some degree of visualisation both in thinking and in communicating. Hand gestures, verbal descriptions and diagrams were the resources that participants drew upon as aids for visualisation. Most were able to explain one or two of the phases (usually the full moon, the new moon or the half moon) but not all of them.

Participants who knew the correct mechanism, but were unable to explain all the phases, were presented with verbal descriptions of the Anchor Situations and they were asked to reason about these situations (Table 2). To illustrate, Asha declared at the beginning of the interview that her earlier explanation was wrong. When asked if she would like to change her drawing, she made a new drawing which explained the different phases. Although the drawing was broadly correct, she was unsure about it. When asked to draw the shape of the phases, she drew the 'false gibbous' shape. At this point Anchor Situation 1a was described to her and it was suggested that it might help her think about the problem. The presentation of the Anchor Situations was taken as marking the beginning of the second segment of the interview.

The Anchor Situations were not presented to Bina and Dinkar, whose new explanation was thought to be satisfactory. For these two participants, the point in the interview when they began to reason about the various shapes on the basis of new diagrams that they had drawn was taken as the demarcation between the first and the second segment of the interview. The Anchor Situations were presented to all others beginning in every case with AS1a. Individual differences in sequence are shown in Table 6. Anchor Situation 1b was presented as a simpler version of 1a when we felt that participants were having difficulty visualising the various positions and appearances in 1a. However, participants did not immediately recognise the equivalence of the two situations with respect to the appearance of the ball. This led to the description of the anthromorphic versions of the Anchor Situations: 2a and 2b. Here participants were able to see more easily that the situations were equivalent. All the participants made drawings of the ball or the face in different positions for one or more situations. The presentation of the Anchor Situations ended when participants were clear about the how the ball (or face) would look in different positions, and when they were clear that the situation was similar to the ESM system. Except Gauhar, all the others asserted that the Anchor Situations helped clarify the mechanism causing the phases of the moon.

Asha	AS1a (15:00) → AS1b (26:00) → AS1b' (34:00) → AS2a (55:00) → AS2b(58:00)
Bina	-
Chander	AS1a (25:00)
Esha	-
Farha	AS1a (19:00)
Farha	AS1a (18:00) → AS1b (26:00) → AS2a (35:00) → AS2b(41:00)
Gauhar	AS1a (28:00) → AS1b (37:00) → AS2a (46:00) → AS2b(51:00)
Harsh	AS1a (17:00) → AS1b (21:00) → AS1b' (29:00) → AS1a'(30:00)

Table 6: Summary of the Anchor Situations presented in the interview

Interview Segment 3: When the participants were clear about the mechanism causing the phases, they were asked to think more carefully about the shape of the various phases – 'what is the exact shape of the boundary in the half moon phase? How do the curves forming the boundary look for the other phases?' Two shapes were focused on especially – the half moon and the gibbous phases. In all the phases, one part of the boundary is the outer edge of the moon and is a semicircle. The inner or 'defining boundary' is a projection of the 'illumination boundary', which is the line separating the lit and the dark halves of the moon's surface. The defining boundary is a semi-ellipse which changes continuously from a semicircle (full moon phase) to a convex curve (gibbous phase) to a straight line (half moon phase) to a concave curve (crescent phase). As mentioned earlier, some participants thought that the defining boundary of the half moon phase was never a straight line but was always curved. For the gibbous phase, some consistently drew a concave, i.e., the 'false gibbous' shape, instead of the correct convex defining boundary (Table 4).

Once the mechanism was clear to them at the end of Interview segment 2, the architects Chander and Dinkar, and the physicist Farha readily concluded that the defining boundary was elliptical (Table 7), although only the two architects could make accurate drawings. Esha and Harsh thought initially that the curve was a circular arc, but later correctly described it as elliptical. The interview transcript shows that Esha had difficulty in visualising the illumination boundary on the moon. Once her attention focused on this boundary, she realised that its projection was an ellipse. For Harsh, the change was preceded by a close examination of the painted black and white ball. Asha thought initially that the curve was concave (false gibbous), but described it correctly after she realised the equivalence of the anchor situations. Two participants – Bina and Gauhar were unable to arrive at a correct and consistent description of the defining boundary, although both had an opportunity to examine the black and white painted ball.

Asha	Bina	Chander	Dinkar	Esha	Farha	Gauhar	Harsh
Concave→ Semi ellipse	Arc of circle	Semi ellipse	Semi ellipse	Arc of circle→ Semi ellipse	Semi ellipse	Unsure	Arc of circle→ Semi ellipse

Table 7: Participants’ description of the defining boundary of the gibbous phase in the interview

Discussion

One of the points of interest in the study was when, how and why participants changed their view about the basic mechanism causing the phases of the moon. All six participants who presented incorrect explanations corrected their explanation – the details are summarised in Table 5. We could identify two factors that induced the change of explanation. The first was the interviewers' questions which led to an awareness about the difference in the durations of the lunar eclipse (a few hours) and the lunar phase cycle (one month). Secondly, the situations described in the hint sheets and the anchor situations were familiar and were recognised as being analogous to the ESM system. Visualising these situations helped participants in arriving at the correct mechanism. It is noteworthy that participants changed to the correct explanation quite easily. Despite this however, the participants found it difficult to complete the two sub-tasks identified in the study: explaining all the phases clearly on the basis of the correct mechanism, and describing the exact shape of the phases.

A major source of difficulty for the participants was to understand that the earth's rotation or the observer's position on the earth have no causal role in the occurrence of the lunar phases, but only determine when and whether the moon is visible at all. Chander unsuccessfully attempted to include the earth's rotation in his mental simulation. Some participants were not sure if the same phase would be visible from different points of the earth, but most of them corrected this when they were reminded about the distance scales. A related but different difficulty was the confounding of the phenomenon of the visibility of the moon and the lunar phases. For example, Esha offered an alternative explanation for the new moon: "if the moon rises and sets below the horizon then ... it will be a new moon". This correctly explains why the winter sun is not visible from the polar regions, but is an incorrect explanation of the new moon. When Bina was requested to draw a top view of the ESM system in addition her original side view, she discovered that the new moon is in the sky at daytime along with the sun, which appeared to confuse her.

These responses may be interpreted as attempts by participants to incorporate salient pieces of factual knowledge such as that the earth rotates, or that the observer is located at a point on its spherical surface – into their reasoning processes. The cognitive demands that arise from attempting to incorporate such information into mental simulations are heavy. One can only solve the task through a reduction of the dynamic aspects. We found three ways in which such reduction was achieved by participants. The first was simplification by ignoring aspects of the problem such as the earth's rotation or the observer's position. Although participants made such simplifications, they did not explicitly justify them by referring to the distance scales. A second reduction strategy was to take snapshots of the dynamic configuration. This was visible in the drawings that participants made showing multiple positions of the moon as it moved in its orbit. An interesting variation of this strategy was the simplification made by Gauhar – he considered ESM configurations at the same time of the night (10 pm) on successive days. This allowed Gauhar to represent the incremental shift in the moon's position each day when the observer looks at the moon from the same position on the earth at 10 pm. A third strategy was of identifying and representing invariants. We shall discuss this in the context of the use of diagrammatic reasoning.

The explanation of the lunar phases also calls for shifting one's perspective from a space based standpoint to an earth based standpoint. Cognitive studies of perspective taking distinguish this from other forms of mental imagery such as mental rotation (Hegarty & Waller, 2004). People find it very difficult to visualise the effect of changes in orientation without kinesthetic feedback (Klatzky et al., 1998). Thinking about the ESM system requires even further radical adjustments besides changes of perspective and orientation – the horizon, which has a salient presence in an earth based perspective, but which disappears from a space based perspective; vast distance scales; absence of landmarks; accounting for dynamic aspects like the earth's rotation, and spatial aspects like the curvature of the earth.

Representing familiar situations that are analogous to the ESM system is one way of meeting the cognitive demands of the task. Indeed, model based reasoning often is analogical reasoning. The anchor situations were helpful to participants in making the required perspective shifts. Some anchor situations were easier than the others either because of a reduction in the dynamic aspects (AS1b) or because they were anthropomorphic (AS2a, 2b). For instance, Harsh had much difficulty in visualising the relative positions of the sphere and the observer in AS1a and 1b. He was then asked about the situation where the partially lit sphere in AS1a was replaced by a white ball painted half-black, with the white half always facing one of the interviewers (Shamin) standing in the distance. Harsh immediately said, “Shamin is sun. Here is the moon, this part is white” and then, “Ya, this last one (situation just described) helps.... With the ball facing Shamin, I can think of this.” (34:00) Farha and Gauhar too found it easier to visualise the partial views in the anthropomorphic situations As2a and 2b. Besides the presence of familiar elements such as faces in these situations, it is easier to visualise the actions of orientation and perspective taking as bodily actions, and to visualise changes in orientation with respect to the reference frames centered in one's own or another person's body (Tversky, 2005).

The anthropomorphic versions were useful also to understand the equivalence of the partial views obtained in AS1a and 1b. Asha, for example, thought that the defining boundary for the gibbous phase was concave for AS1a and convex for AS1b. She suggested that the illumination boundary on the stationary sphere (AS1b) lit by a distant light was like a ring attached on the surface of the sphere. However when the sphere is moving (AS1a) we cannot think of it like a ring. “*It* [the lit portion] *is moving into darkness ... constantly patches of darkness fall on it in parts ...*” At this point we presented Asha with the verbal descriptions of the anthropomorphic versions of the Anchor Situations: AS2a and 2b. On drawing diagrams of these situations, Asha realised that the appearances of the face would be same in both situations (Figure 3). Reasoning back to AS1a and 1b, Asha concluded that the appearances of the sphere would be the same here as well. This led to her correcting the curve of the defining boundary of the gibbous shape. In her own words the hints that helped her were the shadow of the ball in hint sheet 2, the Anchor Situation with the painted ball (AS1b') and finally, the Anchor Situations where she had to visualise the friend's face (AS2a and 2b). The last situations were easy because “*you don't have to refer to any imagery in your head ... the thing is I can imagine the person ...*”. By `imagery' she appears to mean effortful mental visualisation.

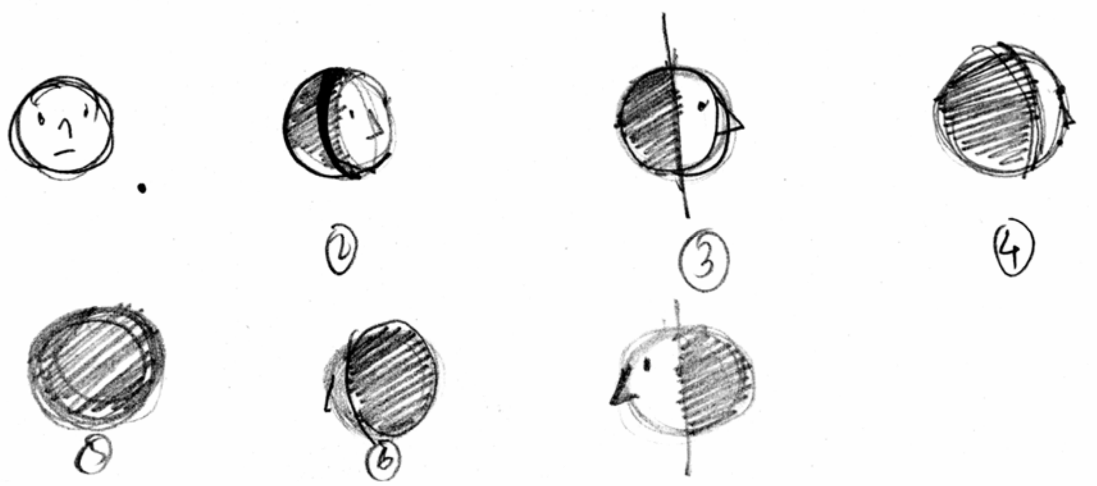


Figure 3: Exploring the appearance of the face in Anchor Situation 2a. (Asha)

Shape of the phases: Three points are critical to completing this sub-task: (i) realising that always half of the moon's spherical surface is lit (ii) focusing on the illumination boundary – and (iii) projecting the illumination boundary on to a two dimensional surface. One surprising view, also reported in Trundle et al. (2007) was that the closer the moon is to the the sun, the larger is its illuminated area. This was as if an inverted cone of light emanated from the sun with its base at the sun (Figure 4a). Textbooks often use such diagrams to mark the umbral and penumbral regions of an eclipse. This view appeared only briefly in the interviews, but was articulated clearly by a participant in the pilot interview. Another misconception, expressed by Bina, was that the light bends around the moon resulting in more than half of it being lit by the sun. This is probably a misinterpretation of the phenomenon of the bending of light rays close to the sun during a solar eclipse. Bina had also exaggerated the size of the sun in her diagram, which suggested that more than half the moon would be lit (Figure 4b).

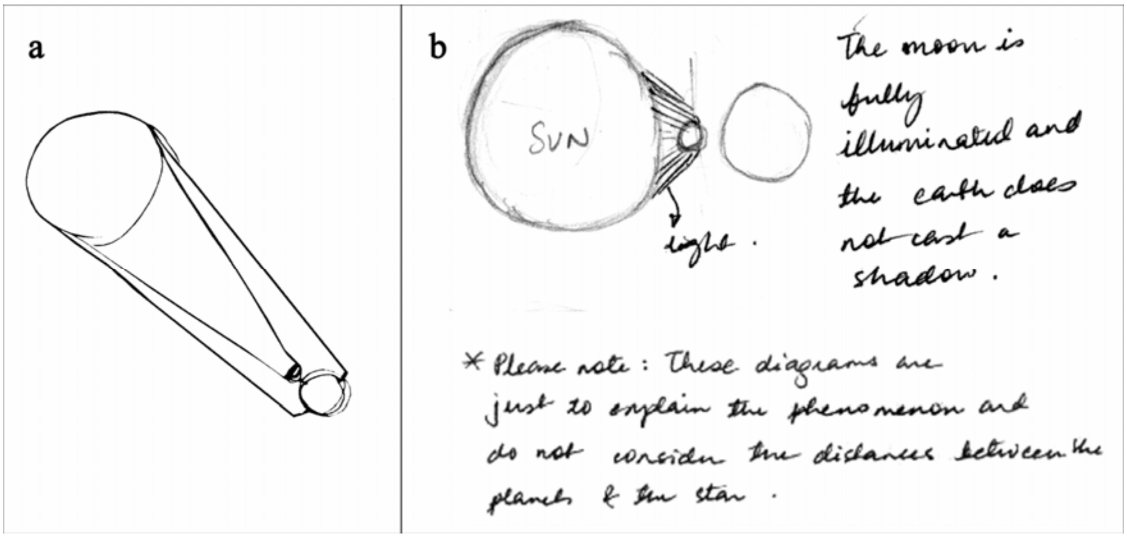


Figure 4: (a) Suggestive of Inverted Cones of light. (b)The sun's rays illuminate more than half the moon. (Bina)

Some participants had difficulty focusing on the illumination boundary. Bina was unsure about the existence of the illumination boundary. She thought that the light would be diffused near the edge of the illuminated hemisphere since the light rays are nearly parallel to the surface. This argument is correct, and the fact is responsible for the exaggerated shadows and the irregular appearance of the illumination boundary of the moon when seen through a telescope. However, given the great distance to the moon, the moon's phase usually has a clearly visible boundary line. This is a helpful, even necessary, simplification in obtaining the precise shapes of the phases. Asha and Esha, for example, needed to represent the illumination boundary clearly before they completed this sub-task. Early in the interview, before Asha focused on the illumination boundary, she resorted to a misleading analogy to account for the 'false gibbous' shape describing it as like a circle with a “bite taken out of it” or an orange with a segment removed. Focusing on the illumination boundary did not immediately lead her to the correct shape. The key was representing the illumination boundary as an invariant in a dynamic situation, which she was able to do only after she visualised the anchor situations, as discussed above. Esha realised with questions from the interviewers that half the spherical surface was lit and half was dark. When she was asked to focus on the boundary line between these two halves and asked how it would look in two dimensions, she at once replied that it would be elliptical. The three physicists, who realised that the illumination boundary would be elliptical (Table 7), had difficulty in deriving or drawing the precise shape of the phases, in contrast to the three architects who immediately projected the circular illumination boundary on the spherical surface onto two dimensions and drew the resultant phases.

Visual Representations: In order to explain the phases in detail, participants needed to generate an external visual representation of a particular configuration of the ESM system, insert the observer into this configuration, and work out how the moon would appear from the observer's perspective. Two physics graduates Gauhar and Harsh, used gestures to generate a representation (Figure 5). Harsh used a pencil in the left hand to represent the moon, and the right hand to represent the sun and the rays emanating from the sun. His eyes and head represented the observer on the earth. However he had difficulty in moving the left hand independently of the motion of the head and the right hand and after a while gave up reasoning with this gestural representation. Gauhar used the left hand to represent the moon and the right hand to represent a variety of other features in a dynamic fashion: position of observer on the earth, direction from which light rays fall, curved surface over which the rays fall, direction in which the light rays were reflected from the moon's surface and so on. Both Gauhar and Harsh were able to explain at best only a particular phase as resulting from a particular configuration and were unable to generalise the explanation to all the phases. Evidently gestural representations were of limited use to represent the ESM system.

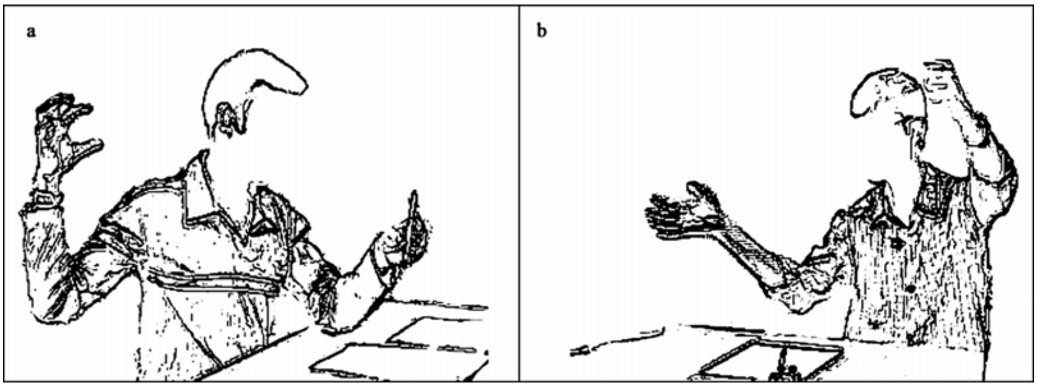


Figure 5: Gestures to model the ESM system (Left – Harsh, Right – Gauhar)

All participants had brief episodes in which they attempted to visualise the configurations without the aid of gestures or drawings. Typically such episodes occurred between gestures or drawings, when the participant was silent and reported or indicated in other ways that she or he was thinking. In the case of Chander, Esha and Harsh, such episodes were long and prominent. They did not lead to clear inferences, but were followed by a verbal question, or an exploration using gesture or drawing. Sometimes one of the interviewers intervened making a suggestion, or presenting a hint situation.

The most productive external representations were drawings, especially when participants introduced key representative elements or transformations in a deliberate manner to extract inferences. Two such examples are in Figure 6, one drawn by Dinkar and the other by a participant J who responded to the pilot questionnaire.

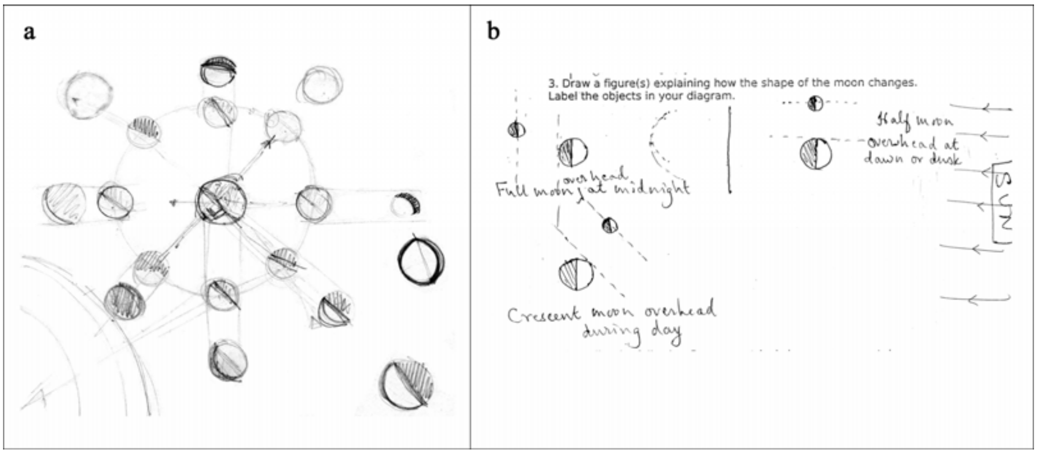


Figure 6: Using diagrams to reason about the moon's phases (i) Dinkar: Observe the projection lines (ii) J (Pilot study participant)

In Dinkar's diagram, the illumination boundary is represented by a diameter across the moon perpendicular to the direction of the sun. Dinkar also drew lines from the earth to represent the limits of the visible portion of the moon. Using a combination of these elements, he was able to project the visible portion of the moon. This mode of visualisation and reasoning was very efficient and Dinkar quickly arrived at a satisfactory explanation of all the phases, concluding that each phase was a shape composed of a semicircle and a semi-ellipse. In J's diagram, the illumination boundary and the limit of visibility (dotted line) are represented as diameters similar to Dinkar's diagram.

The illumination boundary and the visibility boundary are important invariants in the ESM configuration. They are 'abstract' in relation to the material objects that are constantly in motion: the rotating surface of the moon, which is itself revolving around the earth. The relative positions of these lines determine the lit portion of the moon visible from the earth. The illumination boundary is always perpendicular to the direction of the sun and the planes in which the illumination boundaries lie as the moon revolves around the earth are nearly parallel. The plane of the visibility boundary is perpendicular to the direction of the earth. These invariant objects and their properties are the conceptual elements which provide the key to solving the tasks in the study. We hypothesise that conceptual change and learning in the context of this problem involves grasping these invariants. Planning and making diagrams force reduction of a dynamic to a static situation and increase the likelihood of identifying invariants. Further diagrams offer possibilities of transforming representation of invariants to extract inferences (Ramadas, this issue).

Finally, the participants' responses suggest some reasons why the shadow explanation is so widely prevalent. Typical diagrams of the ESM system exaggerate the size of the earth and the moon and under-represent distances. If such large bodies move in close proximity and a source of light is aligned with them, it appears inevitable that the shadow of the earth would fall frequently on the moon. Moreover shadows are a part of everyday experience; whenever there is a strong source of light, there are always shadows. Two dramatic events in the sky – the lunar and solar eclipses – are indeed caused by shadows. It appears parsimonious to assume that this mechanism also causes the phases of the moon. Esha appeared to be expressing this concern when she wondered aloud whether one mechanism is sufficient to explain both eclipses and phases. Further, the study makes it evident that the eclipse mechanism is much simpler to visualise than the correct mechanism. The former requires one only to think of the overlap of 2-dimensional shapes in contrast to the complex reasoning that the latter requires. Some kinds of representations also contribute to the prevalence of the shadow explanation. In a 'top view', with the line of sight perpendicular to the orbital plane, the five degree inclination of the moon's orbital plane to the ecliptic cannot be represented. In this view, it appears that the shadow of the earth must fall on the moon (Figure 1 and 4a). To give a counter-illustration, Chander drew a separate diagram exaggerating the inclination and was able to explain the full moon phase correctly (Figure 7). Gauhar used the following simplification explicitly – he had made the plane of the moon's orbit parallel to the ecliptic, but had raised it above the latter, ensuring the shodow of the earth did not fall on the miin (Table 3).

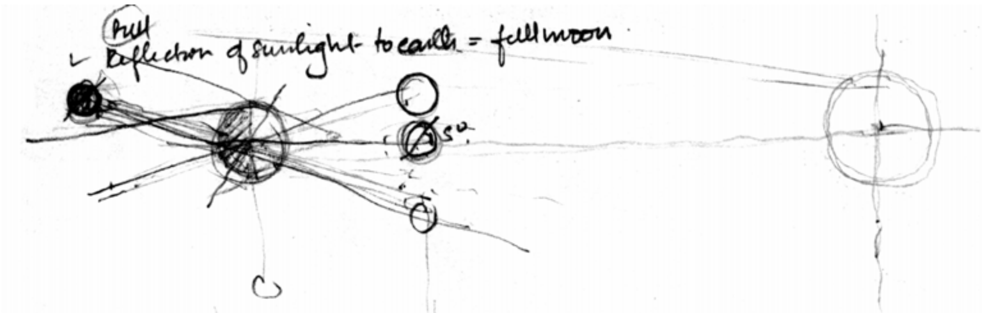


Figure 7: Showing the inclination of the moon's orbit to explain the full moon (Chander)

Another factor in reinforcing the shadow explanation may be the difficulty in projecting curves on the sphere on to two dimensions. This may partly account for the 'false gibbous' shape, which was drawn even by participants who understood the correct mechanism. This incorrect diagram, in turn, might reinforce, or at least recall, the shadow interpretation. At one point in the interview Asha affirmed the concave shape of the defining boundary for the gibbous shape, and this reminded her of a shadow covering the moon. She said, "*it's just that the curve changes ... hmm... shadow .... I don't think it is another circle interposed which is just moving across.*" (23:00) Difficulties in internal and external representations may thus mutually reinforce each other. Educators need to be alert to the many ways in which individuals can be misled in their reasoning.

Conclusion

Although the ESM system can be represented by a simple physical model, the visuospatial reasoning required can be complex. Consequently, although participants in our study had a correct mental model, namely, the orbital model of the ESM system, they advanced incorrect explanations for the lunar phases. Moreover, even after identifying the correct mechanism causing the phenomenon, they were not able to explain all the phases satisfactorily. In order to successfully explain the lunar phases, one needs to shift perspectives as one reasons, from a space based to an earth based viewpoint. Further, one needs to extract invariants from a dynamic situation. These invariants are conceptual elements such as the illumination boundary and the boundary of visibility, and belong to the domain of the geometry of the sphere. Once these invariants are identified, the problem of obtaining the exact shape of the phases reduces to projecting curves on the 3-D surface of the sphere on to two dimensions.

Participants recalled factual, verbal knowledge associated with the ESM system and attempted to integrate this with visuospatial reasoning. For such reasoning to be successful, one needs to make idealisations and approximations, with regard to which motions are relevant to the problem and which are not, what aspect of the observer's location may be ignored, and how one can accommodate the distance scales involved.

Visuospatial reasoning processes need to be implemented on suitable external representations, which need to be selected and generated in the first place. Well chosen and simple diagrams are efficient external representations. Transformations on these diagrams – representing multiple positions of the moon, drawing projection lines, representing invariant elements – allow one to think and reason and draw the appropriate inferences. We have provided illustrations of how diagrams can be powerful tools in visuospatial reasoning. Diagram based reasoning offers promise as a strategy that can be learnt and adapted by students following suitable instruction which includes the elements of representing, transforming and projecting 3-dimensional objects on to two dimensions. Visualisation is also aided significantly by a suitable choice of familiar situations analogous to the target situation. In instruction about the moon's phases, the Anchor Situations (starting from the easiest, in reverse of the order presented in the interview) and the hints described in the hint sheets are likely to be helpful.

The study also illustrates the effectiveness of a methodology where participants are invited to reason and provide detailed explanations of a simple familiar phenomenon. Participants actively grappled for long periods with the problem and found the problem engaging, despite the interviewers' gentle suggestions that the interview could be stopped if they were tired. The hints provided helped them make progress in their reasoning and at the same time elicited rich details about their reasoning processes.

References

Gilbert, J. K. (2005). Visualisation: A metacognitive skill in science and science education. In Gilbert, J. K. (Ed.),Visualisation in science education (pp.9-27). Dodrecht, Springer.

Heath, T.L. (1931). *A history of Greek mathematics 1.* Oxford, Oxford University Press.

Hegarty, M. & Waller, D. (2004). A dissociation between mental rotation and perspective-taking spatial abilities. *Intelligence*, 32, 175-191.

Klatzky, R. L., Loomis, J. M., Beall, A. C., Chance, S. S., Golledge, R. G. (1998). Spatial updating of self-position and orientation during real, imagined, and virtual locomotion. *Psychological Science*, 9(4), 293 – 298.

Magnani, L. (2001). *Abduction, reason and science.* New York, Plenum Publishers.

Nersessian, N. J. (1999). Model based reasoning in conceptual change. In Magnani, L., Nersessian, N.J. And Thagard, P. (eds.) Model Based Reasoning in Scientific Discovery. New York, Kluwer.

Nersessian, N. J. (2002). The cognitive basis of model-based reasoning in science. In Carruthers, P., Stich, S. & Siegal, M. (eds.) The Cognitive Basis of Science. Cambridge University Press.

Ramadas, J. (This issue). Visual and spatial modes in science learning. *International Journal of Science Education*.

Stahly, L. L., Krockover, G. H., Shepardson, D. P. (1999). Third grade students' ideas about the lunar phases. *Journal of Research in Science Teaching*, 36 (2), 159 – 177

Trundle, K. C., Atwood, R. K., Christopher, J. E., (2002). Preservice elementary teachers' conception of moon phases before and after instruction. *Journal of research in Science Teaching*, 39(7), 633 – 658.

Trundle, K. C., Atwood, R. K., Christopher, J. E., (2007). A longitudinal study of conceptual change: Preservice elementary teachers' conception of moon phases, *Journal of Research in Science Teaching*, 44 (2), 303-326.

Tversky, B. (2005). Visuospatial reasoning. In Holyoak K. and Morrison R. (Eds.), The Cambridge Handbook of Thinking and Reasoning (pp. 209-240). Cambridge, MA: Cambridge University Press.

Vosniadou, S. & Brewer, W. F. (1992). Mental models of the earth: A study of conceptual change in childhood. *Cognitive Psychology*, 24, 535 – 585.