Classroom

In this section of Resonance, *we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.*

Pitfalls in Elementary Physics – 3. Work and Energy

The work-energy theorem – a scalar consequence of Newton's II law of motion – for a single particle is simple. Work done by a net force on a particle equals change in its kinetic energy. For a finite (deformable) body – a system of particles – the form of the theorem is essentially the same, but now since internal forces exist in the system, several non-trivial difficulties arise due to the mixed roles of external and internal forces. We illustrate the difficulties by raising some questions that could occur to any critical student.

Does Internal Work (i.e. work done by internal forces in a system of particles) add up to Zero by Newton's III Law?

This is a seductive trap – a pseudo III law – that we must beware of. According to the III law, the internal forces do indeed cancel pair-wise so that the total internal force in any system is always zero. But that is not true about work. Take two masses at rest connected by a compressed spring and release the spring. The internal forces add up to zero, as always. But they are doing positive work. If there is no external force, the system as a whole does not move i.e. its centre of mass is stationary. But work done by internal forces (internal work) is non-zero; it goes up to increase the internal kinetic energy of the system. A body

Arvind Kumar
Homi Bhabha Centre for Science Education, Tata Institute of Fundamental Research
V N Purav Marg, Mankhurd
Mumbai 400088, India
email:arvindk@hbcse.tifr.res.in
Fax: (022) 556 6803

falling under gravity is another example. Force on the body by the earth is always equal and opposite to the force on the earth by the body (III law). But the two work quantities are not equal and opposite. In fact work done on the body by the earth has the same sign (positive) as the work done on the earth by the body, since the displacement is each case is in the same direction as the force. As for magnitude, the latter is nearly zero since the displacement of the earth is negligible. Internal work happens to be zero for a rigid body. But for deformable bodies, systems of rigid bodies or a general system of particles, internal forces can do work (see *Box 1*). The simple reason is (as seen in the examples cited already) that though forces are pair-wise equal

---

Box 1. Internal Work

If $\overline{F}_{ij}$ is the force on particle i due to particle j, by Newton's III law, $\overline{F}_{ij} = -\overline{F}_{ji}$.

This implies

$$\sum_{i,j}{}' \overline{F}_{ij} = 0 \;,$$

since the sum involves pairs of forces that add up to zero. (The prime on summation sign means that $i = j$ terms are to be excluded in the sum). The total work done by internal forces is

$$\Delta W^{\text{int}} = \sum_{i,j} \int \overline{F}_{ij} \cdot \mathrm{d}\bar{r}_i$$

[The limits of the integral (not shown) correspond to initial and final configurations of the system.]

Internal work does not, in general, vanish since $\mathrm{d}\bar{r}_i$ may differ from one particle to another and cannot be taken out of the summation. ΔW^{int} is zero if displacements are common to all particles. For a rigid body, the distance between any two particles is fixed. That is $\bar{r}_{ij} = \bar{r}_i - \bar{r}_j$ has a fixed magnitude, implying $\mathrm{d}\bar{r}_{ij} = \mathrm{d}\bar{r}_i - \mathrm{d}\bar{r}_j$ must be normal to $\bar{r}_{ij}$. Consequently,

$$\Delta W^{\text{int}} = \sum_{i,j}{}' \int \overline{F}_{ij} \cdot \mathrm{d}\bar{r}_i = \sum_{i,j}{}' \int \overline{F}_{ji} \cdot \mathrm{d}\bar{r}_j = -\sum_{i,j}{}' \int \overline{F}_{ij} \cdot \mathrm{d}\bar{r}_j = \frac{1}{2}\sum{}' \int \overline{F}_{ij} \cdot \mathrm{d}\bar{r}_{ij} = 0$$

assuming internal forces are along $\bar{r}_{ij}$, the line joining the particles i and j.
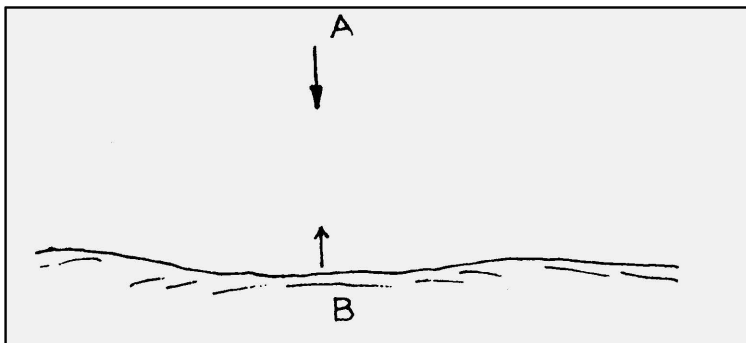
Figure 1 *Students some-times employ a wrong version of the III law: work done on A by the force due to B is equal and opposite to the work done on B by the force due to A. How flawed the idea is can be seen in the simple example of a body falling to the ground.*

and opposite, the displacements of particles in a·pair may be different in magnitude, direction or both. In short, for a pair of bodies A and B, $\overline{F}_{AB} = -\overline{F}_{BA}$ by the III law, but the work quantities W_{AB} and W_{BA} bear no such simple relation, in general.

In some situations, however, they do. Consider the quasi-static expansion of a gas in a cylinder with a movable piston. Here the work done by the gas on the surroundings is the negative of the work done on the gas by the surroundings. Why?

What drives a car on a horizontal road: fuel inside the car or friction outside between the road and the tyres?

We all know the answer, that is until we become physicists! Then we get into problems. A point particle will move from rest or change its velocity only if there is a net external force on it; and *it changes its energy only if the external force does work on it.* For a system of particles, the first part of the statement is true if applied to its centre of mass. The centre of mass of a system will not move from rest (or, accelerate, in general) unless there is a net external force on the system. But the second part of the ·statement (italicized) is not always true for a system of particles. For a car (i.e. its centre of mass) to move from rest or accelerate, friction (the only possible net external force in the situation) is necessary. But friction does no work since, for perfect rolling of the wheels, the point in instantaneous contact with the road is stationary. (In practice, friction does negative work.) We thus have an apparently paradoxical situation that while external force (friction) is necessary to accelerate the centre of mass, the
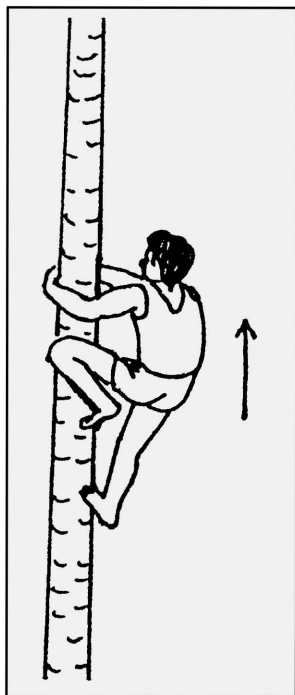
Figure 2. A man climbing a pole with increasing speed. The net external force (average) on the man is upwards, but the work done by the external forces (friction and gravity) is negative. It is the internal work that more than compensates for the negative external work to provide for the increasing kinetic energy of the climber. The climber needs an anchor (pole to grasp by his hands) to provide for upward external force but does not need external energy. He expends his internal source of energy that results after a while in a feeling of tiredness.

car picks up kinetic energy not due to the work of the external force but due to internal work. This internal work needs to be supplied by some energy source: of chemical origin (fuel) in an ordinary car or of mechanical origin (spring) in a toy car. It all looks messy, but it is not if we look at the work-energy relations for a system of particles carefully (see *Box 2*).

Using *Box 2*, we can avoid a number of possible pitfalls:

First, the roles of external and internal forces do not separate neatly. We might think in a simple-minded fashion that work done by external forces changes the kinetic energy of the centre of mass and that by internal forces changes the kinetic energy of relative motion i.e. kinetic energy in the centre of mass frame. This is, in general, a flawed idea as is evident from (7) and (10) of *Box 2*. (The idea works only when $\Delta W'^{ext} = 0$). *Box 3* gives a simple situation where such a separation is true, while *Box 4* gives another simple example where it is not true.

Second, we must be careful which work quantity we are talking about. Total work done by a number of forces on a system of particles is not the same thing as the work done by the total (net) force. This is why total work done by internal forces is not necessarily zero even though internal forces always sum to zero. In the same manner, total work done by external forces is not, in general, equal to the work done by the total external force on the centre of mass. The latter equals the change in the kinetic energy of the centre of mass. There is an additional term in the former quantity representing the work done by the external forces in the centre of mass frame ((7) in *Box 2*).

Two consequences of these observations merit attention. First, if the total external force is zero, it does not mean 'external work' (i.e. work done by external forces) is zero. All it means is that the kinetic energy of the centre of mass does not change. The kinetic energy of the system may, however, change because of the change in the kinetic energy of relative motion. This is precisely the situation in *Box 4*. Conversely, if external work is

Box 2. Work-energy Relations for a System of Particles

For a system of particles, total work done is the sum of work done by the force on each particle

$$\Delta W = \sum_i \int \overline{F}_i \cdot d\overline{r}_i \qquad (1)$$

Now force on a particle is made of an external force and the sum of internal forces due to all the other particles:

$$\overline{F}_i = \overline{F}_i^{\text{ext}} + {\sum_j}' \overline{F}_{ij} \qquad (2)$$

Thus the total work splits into two parts, external work, ΔW^{ext} and internal work, ΔW^{Int}:

$$\Delta W^{\text{ext}} = \sum_i \int \overline{F}_i^{\text{ext}} \cdot d\overline{r}_i \qquad (3)$$

$$\Delta W^{\text{int}} = {\sum_{i,j}}' \int \overline{F}_{ij} \cdot d\overline{r}_i \qquad (4)$$

Applying work-energy theorem for each particle in (1), ΔW equals the change in total kinetic energy (ΔT) of the system. Thus

$$\Delta T = \Delta W^{\text{ext}} + \Delta W^{\text{int}} \qquad (5)$$

If the centre of mass is located at $\overline{R}$, $\overline{r}_i = \overline{R} + \overline{r}_i'$, wherer $\overline{r}_i'$ is the position vector of particle i relative to the centre of mass. Then ΔW^{ext} can be written as

$$\Delta W^{\text{ext}} = \int \overline{F}^{\text{ext}} \cdot d\overline{R} + \sum_i \int \overline{F}_i^{\text{ext}} \cdot d\overline{r}_i' \qquad (6)$$

where the first term represents the work done by the total external force on the centre of mass and the second is the total work done by external forces in the centre of mass frame. This gives

$$\Delta W^{\text{ext}} = \Delta T^{\text{cm}} + \Delta W'^{\text{ext}} \qquad (7)$$

where ΔT^{cm} is the change in the kinetic energy of the centre of mass. A similar calculation yields

$$\Delta W^{\text{int}} = \Delta W'^{\text{int}} \qquad (8)$$

Now

$$\Delta T = \Delta T^{\text{cm}} + \Delta T' \qquad (9)$$

where $\Delta T'$ is the change in kinetic energy of the system relative to the centre of mass frame. Equations (5), (7), (8) and (9) give

$$\Delta T' = \Delta W'^{\text{ext}} + \Delta W'^{\text{int}} \qquad (10)$$

The form of the work-energy relation remains unchanged as we go from the lab frame (inertial) to the centre of mass frame that may be non-inertial. This is a non-trivial result that follows from the characteristic nature of the centre of mass frame [2].

Box 3
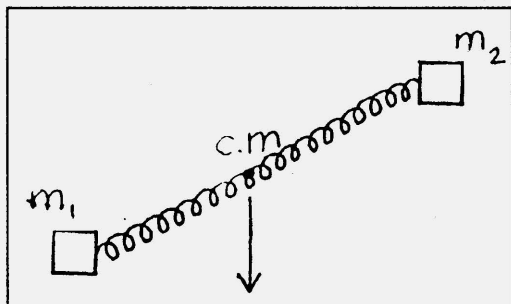
Two masses connected by a spring falling under gravity.

Consider two masses connected by a compressed spring. Let it fall under gravity as the spring is released. Here

$$\Delta W'^{\text{ext}} = \int m_1 g \hat{k} \cdot d\vec{r}_1' + \int m_2 g \, \hat{k} \cdot d\vec{r}_2'$$

$$= g\hat{k} \cdot \int d \, (m_1 \, \vec{r}_1' + m_2 \, \vec{r}_2') = 0$$

where $\hat{k}$ is the direction of gravity. Therefore, from (7) and (10) of *Box 2*, $\Delta W^{\text{ext}} = \Delta T^{\text{cm}}$ and $\Delta W^{\text{int}} = \Delta W^{\text{int}} = \Delta T'$.

Thus there is a clear separation between the roles of external and internal forces: the external force changes the centre of mass energy and the internal forces change the internal energy (energy in the centre of mass frame).

zero it does not mean total external force is zero. Thus, the kinetic energy of the centre of mass may change though no work is done externally. In this case, it is the internal work that changes the kinetic energy of the centre of mass as well as the kinetic energy of relative motion. This is the situation of a car accelerating on a road, with its wheels perfectly rolling. In some examples, such as a man climbing a pole, work done by the external forces is negative. Here internal work must compensate

Box 4

Take the same mass-spring system as in *Box 3* and compress it quasi-statically by two equal and opposite external forces. At each mass the external force is balanced by the internal spring force. The total external force is zero. For this case,

$$\Delta W^{\text{ext}} = \Delta W^{\text{ext}}, \qquad \Delta T' = 0, \qquad \Delta T^{\text{cm}} = 0,$$

which implies $\qquad \Delta W^{\text{ext}} = -\Delta W^{\text{int}}$.

If the spring force is conservative $- \Delta W'^{\text{int}}$ is the change in the potential energy of the spring. Thus the work of the external forces changes the internal potential energy of the system – a familiar idea.
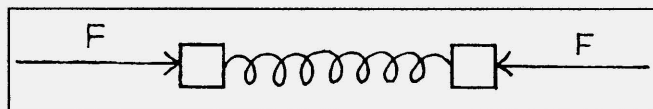
A spring compressed by two equal and opposite forces at its ends.

for it and further provide for any increase in the average kinetic energy of the climber.

Box Sliding on a Rough Horizontal Table: What is the Work Done by the Force of Friction (f)?

A most likely response to this question is: $-f \times d$ where d (positive) is the distance covered by the centre of mass of the block (–ve sign because frictional force is opposite to the displacement). In fact the answer is only 'formally' correct. Remember, friction is distributed over the interface, which is not a plane smooth surface at smaller scales; the origin of friction is a difficult business to analyse. Thus the actual work done by friction cannot be calculated, except perhaps in simplified models. Then what does the work quantity f x d stand for?

Let us first apply the II law to the block alone. We get

$$(F - f)d = \frac{1}{2} m (v_f^2 - v_i^2)$$

where d and $v_f(v_i)$ are the displacement and final (initial) speed of the centre of mass of the block. Next apply the I law of thermodynamics to the 'block plus table' system. If Q represents loss of heat to the surroundings, we have

$$F \times d = \frac{1}{2} m (v_f^2 - v_i^2) + \Delta U + Q$$

where ΔU is the change in internal (thermal) energy of the 'block plus table' system. (This is what causes a slight rise in temperature.) These equations give

$$f \times d = \Delta U + Q$$

Thus the work $f \times d$ is correctly interpreted as that part of the external work ($F \times d$) that does not go to increase the energy of the centre of mass but gets 'dissipated' to increase the internal energy of the system and give away heat to the surroundings. In ordinary parlance, internal energy increase and heat are used synonymously (unfortunately). Hence the usual statement: 'frictional work is converted into heat'. Notice, we applied the
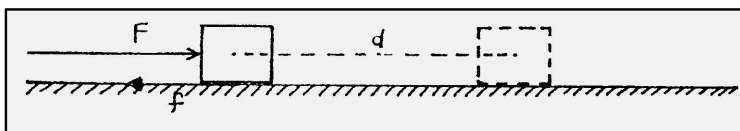
conservation of energy law to the combined system, not to the block alone. Why? Precisely because, as already mentioned we have no clear clue as to the amount of actual work done by the force of friction. ([1] discusses this example in greater detail.)

Finally, it is not only the work-energy relations for finite-sized bodies that can cause confusion. The II law of motion for a system of particles also needs to be carefully assimilated. Though it is a standard textbook matter, we often overlook the fact that what appears like the II law of motion for a system of particles

$$\overline{F}^{\text{ext}} = \frac{d\overline{p}}{dt}$$

is in fact, based on both the II and III laws of motion for the particles constituting the system. The rotational analogue of the II law

$$\overline{N}^{\text{ext}} = \frac{d\overline{L}}{dt}$$

is again based on both the II & III laws of motion for the constituent particles. [The translational II law uses the III law in weak form ($\overline{F}_{12} = -\overline{F}_{21}$) while the rotational II law uses the III law in strong form ($\overline{F}_{12} = -\overline{F}_{21}$, each along the line joining the particles).] There is another thing that needs to be mentioned. In special relativity, we know the law $\overline{F} = m\overline{a}$ is not valid, but

$$\overline{F} = \frac{d\overline{p}}{dt}$$

continues to be correct where $\overline{p} = m\overline{u}$, m being the moving mass of the particle with velocity $\overline{u}$ Somehow, this changing mass context leads many students to believe that the law

$$\overline{F} = \frac{d\overline{p}}{dt} = \frac{dm}{dt}\overline{u} + m\frac{d\overline{u}}{dt}$$

applies universally to any variable mass problem. This is not true. For example, for a rocket that is losing mass due to its

exhaust, the equation above does not apply, whereas for a raindrop falling under gravity, accreting all the mist it encounters, this simple equation turns out to be correct. These statements follow non-trivially by applying the laws of motion to each situation in detail.

Acknowledgements

Suggested Reading

[1] Arvind Kumar and S Barve. *How and Why in Physics.* Vols I to III, HBCSE, 1993–94.
[2] Arnold B Arons. *A Guide to Introductory Physics Teaching.* John Wiley, 1990.

What Mad Pursuit

Had Jim and I not succeeded, I doubt whether the discovery of the double helix could have been delayed for more than 2 or 3 years.

In some ways the code embodies the core of molecular biology, just as the periodic table of the elements embodies the core of chemistry, but there is a profound difference. The periodic table is probably true.

— *Francis Crick*