In this section of Resonance, *we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.*

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Pitfalls in Elementary Physics [1]

2. Newtonian Relativity

Newtonian relativity is often regarded as a simple topic, based on common sense, in contrast to Einstein's relativity which is highly counter-intuitive. While the latter, of course, poses great problems of comprehension, misconceptions abound in Newtonian relativity too, even among good students.

Frames of References

We can associate a frame of reference with any body – the earth, a moving train, etc. We imagine three thin rigid rods extending from some common point to infinity in three non-coplanar (mutually perpendicular, for convenience) directions, moving with the body. To time an event, we can imagine a clock at each point. The clocks are synchronised and co-moving with the body. A frame of reference is clearly an abstract artifact and it is wrong to think that it is limited by the spatial extension of the body or that it 'terminates' at the boundary of the body. Yet, this is how many students think when the following question is posed.

Consider the frame of reference associated with a rotating turntable (Figure 1). What are the forces on (i) a coin on the turntable co-rotating with it and (ii) a coin lying stationary on the ground, outside the turntable? That the frame of reference in question is non-

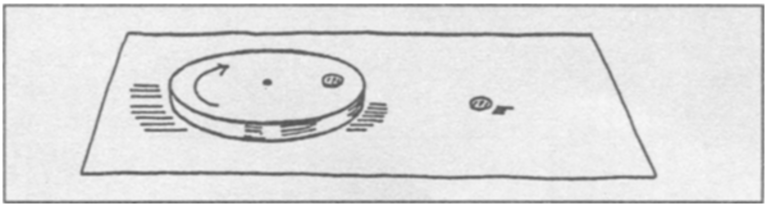
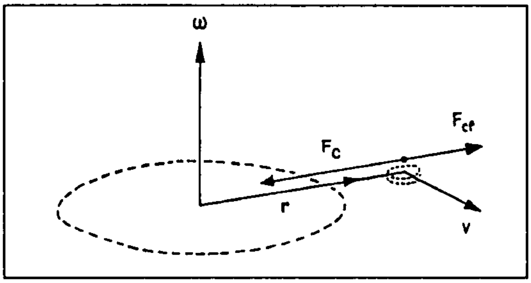


Figure 1. Are there pseudo-forces (centrifugal and Coriolis) acting on the coin on the ground in the rotating frame of reference of the turntable? Students familiar with the notion of pseudo-forces in a non-inertial frame hesitate to say 'yes' to the question, showing the tendency to localise frames by the 'boundaries' of the associated objects.

inertial would generally be known. Most good students would also know that relative to a non-inertial frame, pseudo-forces (centrifugal and Coriolis force on the coin) act on bodies. Thus part (i) of the question would usually evoke a correct reponse: the forces of the coin are its weight and normal reaction of the turntable and the friction and centrifugal force. Each pair of forces adds upto zero, since the coin is given to be stationary relative to the turntable . (For some reason, there is no Coriolis force on the coin.) Very few students can, however, handle part (ii) of the question properly. A common response would be that relative to the turntable's frame, there are no pseudo-forces on the coin on the ground that it is 'outside' the turntable. Frames in this view are thus being localised by the spatial extension of the associated objects – a flawed conception in Newtonian relativity (though in its more subtle form it does appear in general relativity). The correct response to part (ii) of the question is: besides its weight and normal reaction which cancel off each other, the coin is subject to both centrifugal and Coriolis forces. There is no friction here since there is neither impending nor actual relative motion between the coin and the ground. It turns out that the inward Coriolis force is twice in magnitude to that of the outward centrifugal force. There is then a net inward force which, according to the II law, is necessary to keep the coin on the ground moving in a circle relative to the turntable's frame of reference (see *Box 1*).

A more naive confusion is also common among beginners. In physics, a phenomenon is a succession of events which stand apart from the frames of reference used to describe them. This clear separation between phenomena and frames is often not respected by students. Many would think that a child playing

Box 1. Force on a Coin Resting on Ground (Relative to a Turntable's Frame of Reference)



A turntable rotates with constant angular velocity ω. Relative to this rotating frame of reference, a coin resting on the ground will be seen to move in a circle with a velocity $\mathbf{v} = \mathbf{r} \times \omega$ where $\mathbf{r}$ is the position vector of the coin as shown. According to the II law, we need a centripetal force of magnitude $mr\omega^2$ to account for this motion. There is no material force causing this motion, so we must account for it by the pseudo-forces (centrifugal and Coriolis) which appear in a non-inertial frame. The centrifugal force in a rotating frame is given by $\mathbf{F}_{cf} = m\omega\ (\mathbf{r} \times \omega)$ which clearly points away from the centre. The Coriolis force in a rotating frame is given by $\mathbf{F}_c = -2m\omega \times \mathbf{v}$. For the coin, $\mathbf{F}_c = -2m\omega \times (\mathbf{r} \times \omega)$ which points towards the centre and is twice the centrifugal force in magnitude. The resultant of the two pseudo-forces is thus the required centripetal force.

on the deck of a sailing ship is a phenomenon that 'belongs' to the ship's frame of reference, though it can be 'watched' by an outside observer. By the same token, many would hesitate to take the origin of the ship's frame of reference outside the ship ! Once again, the frame is being wrongly equated to the concrete object with which it is associated.

Length and Time Invariance

If the frame S′ moves with uniform velocity $\mathbf{v}$ relative to S, their space-time co-ordinates in Newtonian relativity are related by

$$t' = t + t_0$$
$$\mathbf{r}' = \mathbf{r} - \mathbf{v}t + \mathbf{r}_0$$

For simplicity, we can take $\mathbf{r}_0$ and t_0 to be zero so that the clocks in S and S′ are synchronous and the origins of the two frames coincide at $t = t' = 0$.

These relations (known as Galilean transformations) are based on our intuitive notions of space and time. The time-interval between any two events is absolute, independent of the frame of reference. The displacement between any two events is given by $\mathbf{r}'_2 - \mathbf{r}'_1 = \mathbf{r}_2 - \mathbf{r}_1 - \mathbf{v}(t_2 - t_1)$.

Neither the magnitude nor the direction of the displacement vector is an invariant except when the events are simultaneous. The exception corresponds to the situation for measurement of length of an object. Hence length (distance between simultaneous events) is an invariant in Newtonian relativity.

Surprisingly, these 'obvious' notions of length and time invariance are widely misunderstood. Detailed studies on students' cognition show that the velocity addition law connecting velocity of a body in the frames S and S′

$$\mathbf{u}' = \mathbf{u} - \mathbf{v}$$

that follows from the Galilean transformations is usually respected. But in many situations students readily forsake time invariance to save distance invariance, as the following example shows.

A stream flows with a uniform velocity of 5ms⁻¹. An observer S on a bank sees a boat travel perpendicular to the stream and cover the distance of 1 km from the bank to the opposite bank in 15 minutes. Consider another observer S′ floating along the stream. What is the width of the river for S and S′? What is the distance travelled by the boat relative to S′? What are the times of travel relative to the two frames? (Figure 2).

For S, the distance travelled by the boat is just the width of the stream. Now since the width of the stream is an invariant, a common response is that the distance relative to S′ is also the same as for S, that is 1 km. The velocity transformation using the parallelogram law is very often correctly related. Yet the visual representation of distance by width in the first case is so strong that students would divide the same distance (1 km) by two different velocities, arrive at two different times of travel and fail to be shocked by the answer. A similar response of correctly transforming velocities, taking distance invariance for

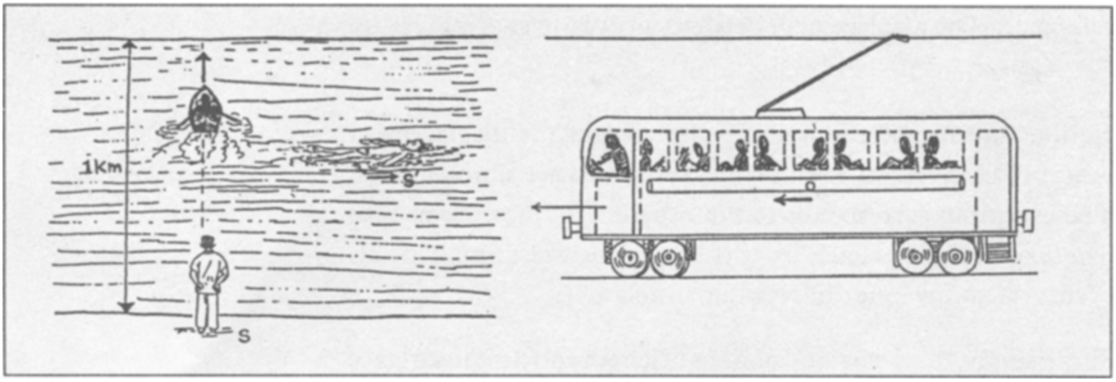


Figure 2. Two simple problem situations (see text) where many students readily violate time invariance of Newtonian relativity to save a supposed distance invariance even for non-simultaneous events.

granted and violating time invariance appears in the following problem. *There is a 10 m long tube along the length of a tram moving uniformly with speed 10 m/s. A ball is rolled down the tube frame from one end to the other with a speed of 1 ms^{-1} relative to the tram. For an observer on the ground, what is the distance travelled by the ball and how much time does it take? (Figure 2).* The frequent answer: (10/11)sec to the last question shows that for physics undergraduates, invariance of time interval is not as obvious as we think – they can abandon it for the more seductive 'distance invariance', even when the concerned events are not simultaneous.

Energy Conservation

Perhaps because it is greatly emphasised, the law of conservation of energy is so sacrosanct to students that they sometimes go to any length to 'save' it even where it is irrelevant. Consider the following question:

For an observer on the ground, the various objects around, trees, buildings and mountains beyond, are at rest i.e. they have zero kinetic energy. The same objects when viewed relative to a moving train's observer acquire huge kinetic energies (because of their large mass). What accounts for this increase in energy?

This question is silly to those who are aware that conservation of energy means that the total energy of an isolated system in a *given frame* does not change in time. Kinetic energy is obviously not invariant between frames in relative motion since velocities do transform and kinetic energy is proportional to the square of the velocity. Thus the difference in kinetic energy of a body in the two frames is kinematic in origin and is not to be accounted for by any dynamical cause. It is, of course, possible to account for the change in kinetic energy of a body from S to S′ as the work done by the pseudo-forces in the (non-inertial) accelerated frame that is co-moving with S initially and with S′ finally. But since pseudo-forces are kinematic in origin, this is not a dynamical explanation of the change in kinetic energy.

A frequent response to this question is something like this: If there is a change in kinetic energy of a body, we must look for a source to account for the change. May be the energy spent in moving the train from rest to its final velocity 'appears' as kinetic energy of the objects outside! That such an absurd response can be so frequent is intriguing. It simply shows students' fixation to the idea of conservation of energy. In our instruction, we need to caution that invariance of a quantity in time (relative to a fixed frame) must not be confused with invariance across frames in relative motion. Conservation of total energy, linear momentum and angular momentum of an isolated system are invariances of the former kind. Acceleration is unchanged (in Newtonian relativity) for frames in uniform relative motion. It is an invariant of the second kind. Total electric charge of an isolated system is an invariant of both kinds.

Inertial and Non-inertial Frames

A frame is inertial if the law of inertia holds good relative to it. Relative to an inertial frame, Newton's second law holds good in its simple form i.e. without any pseudo-forces. If a frame S is known to be inertial, any other frame S′ moving with uniform velocity relative to S is also inertial. If, on the other hand, S′ accelerates or rotates i.e. has non-uniform motion relative to S, it is non-inertial. Inertial or non-inertial character of a frame is clearly an intrinsic property of the frame of reference in question. If one thinks all this is very simple, try the following question on a class of physics undergraduates.

Relative to an inertial frame S, two other frames S′ and S″ move with a common acceleration ***a*** *but with different velocities. S′ and S″ are, therefore, in uniform relative motion (Figure 3). Are they inertial with respect to each other?*

This last question is nonsense, for the phrase 'inertial with respect to some other frame' is meaningless. Yet a great majority of students are likely to say 'yes'. These same respondents are also likely to say that just as S′ is non-inertial with respect to S, S is non-inertial with respect to S′ since it is moving with acceleration $-\boldsymbol{a}$ relative to S′. Inertial or non-inertial character is being treated here as a relative property of frames which satisfies reciprocity – a completely flawed but widespread notion.

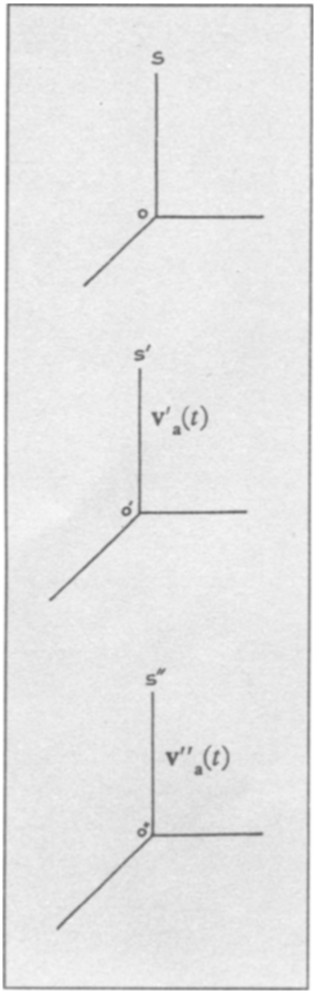


Figure 3. ***S′ and S″ have common acceleration relative to an inertial frame S. Since S″ has uniform motion relative to S′, they are regarded as 'inertial with respect to each other,' a flawed but widespread notion.***

Anthropomorphism

Many elementary books on relativity caution the readers against giving an anthropomorphic connotation to the term 'observer' in physics. Yet the word 'observer' continues to evoke among us the image of a human sitting at the origin of a frame watching the phenomenon! Consider the following question:

An airplane is flying high in the sky with its meters showing a speed of 1000 km per hour. To an observer on the ground it appears to fly rather slowly. In comparison, a bird in the sky appears to fly much faster. Is the speed of the bird greater than that of the plane relative to the ground's frame of reference? Is the actual speed of the plane much greater than the speed of the bird? Most students understand the situation here. The speed of an object as perceived by our eye is the rate at which the line of sight rotates, which is the linear speed of the object divided by its distance from the eye. A bird flying with a much smaller speed say 20 km/hr which is one fifth the speed of the plane, will 'appear' to move much faster if the plane's altitude is much more than 50 times the height at which the bird is flying. Despite this understanding, many students are likely to say 'yes' to the first question, since they equate physical description in a frame of reference to viewing by a human observer.

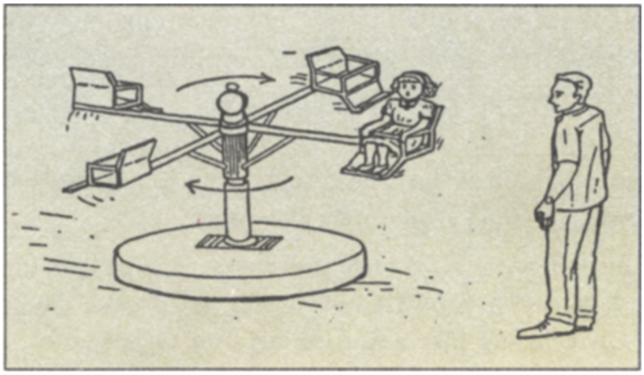


Figure 4. Force is often equated to the 'feeling' of force . A student with this anthropomorphic frame work is likely to assign a centrifugal force to the child in the merry-go-round and deny the same to the man outside, no matter which frame of reference is under consideration.

Anthropomorphism is a source of another serious error; equating forces to 'feelings' of forces. Is there a centrifugal force on a child in a merry-go-round relative to the ground's frame of reference (*Figure 4*)? A common thinking will be something like this: the child does feel pressed outward, so there is a centrifugal force on her. Is there a centrifugal force on a man on the ground relative to the merry-go-round's frame of reference? 'No', many would think, or else wouldn't the man 'feel' the force?

In physics, both answers are wrong. The ground's frame is (approximately) inertial. Taking it, for simplicity, to be perfectly inertial, there is no question of a centrifugal force on any object, since centrifugal force, a pseudo-force, is to be invoked only in a non-inertial frame. Likewise, the merry-go-round's frame is non-inertial, so there is certainly a centrifugal force (as also a Coriolis force) on the man. We need to emphasise in our instruction that the 'feeling' of force by a human arises only when there is an impending relative motion between different parts of our body and our muscles generate suitable strains to check that motion. A man freely falling under uniform gravity has no impending motion between different parts of his body and, therefore, no feeling of force. In any frame of reference, the net external force on a person should not be naively equated to the force 'felt' by the person.

Overall, the examples above show that in physics teaching, it is wrong to dismiss Newtonian relativity as obvious. The naive intuitive notions of the beginners need to be carefully changed to the correct repertoire of prescriptive notions of Newtonian relativity. All we can say is that this transition is not as difficult as that required for Einstein's relativity.

For consolidation of the ideas discussed here, students may like to attempt the following simple exercises:

❖ A large ship is moving uniformly on the sea. An aircraft takes off from the ship and flies out into the distance. After completion of its mission, the aircraft returns to the ship. Is the trajectory of the aircraft in the frame of reference of the ship a closed curve?

❖ In the earth's frame of reference, the sun moves across the sky each day from east to west. Explain what forces cause this observed motion.

❖ In a lift that is going up with uniform speed, a boy can jump up to a maximum height of one metre above the floor of the lift. If, instead, the lift were moving down with uniform speed, to what maximum height above the floor of the lift could the boy jump?

❖ An experiment with a conical pendulum is set up in the laboratory. The motion of the bob is observed by three persons. O 1, viewing the motion from a point directly above the point of suspension, observes the bob to move along a circle. O2, viewing it in the plane of motion of the bob, sees it moving along a straight line. O3, viewing the motion obliquely, finds the trajectory to be somewhat oval-shaped. What is the correct description of the trajectory relative to the laboratory frame of reference?

Acknowledgements

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Suggested Readings

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