Experimental determination of unknown masses and their positions in a mechanical black box

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Abstract
An experiment with a mechanical black box containing unknown masses is presented. The experiment involves the determination of these masses and their locations by performing some nondestructive tests. The set-ups are inexpensive and easy to fabricate. They are very useful to gain an understanding of some well-known principles of mechanics.

Introduction

In different experimental set-ups occurring in physics, a 'black box' refers to a system that acts as a single unit but may be comprised of components that cannot be seen or accessed individually. This unit may be part of a larger system, and can contain electronic, electrical, magnetic, optical or mechanical devices or components, or a combination of them. The idea behind a black box experiment refers to an approach where instead of going into every detail of a particular part, the whole thing is treated as a single unit whose overall performance can be assessed.

Black boxes that contain essentially mechanical components may be explored through mechanics experiments that are normally referred to as mechanical black box (MBB) experiments. The basic idea behind any black box experiment remains essentially the same, i.e. by performing some nondestructive experiments one tries to find out what items are inside and how they are arranged. These types of box are quite interesting and have been introduced into the experimental sections of international competitions such as the International Physics Olympiad (IPhO) [1, 2]. MBBs may contain springs and masses at particular locations or other suitable elements whose characteristics, such as the spring constant, mass and location, can be determined. Here we shall describe an experiment with a particular type of MBB containing two masses.

Design of a black box containing a pair of unknown masses

The MBB may have different shapes and sizes depending on the experimental requirements. The MBB that has been devised and used to perform several experiments is described below.

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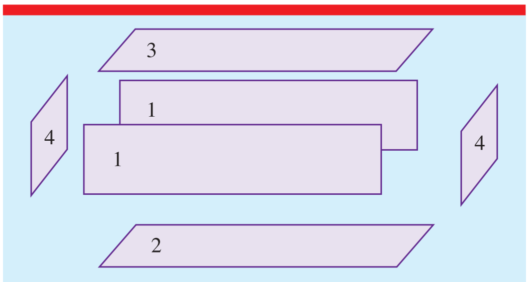


Figure 1. Different parts of the empty box.

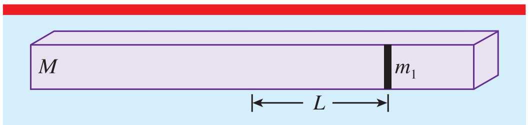


Figure 2. Black box with one mass (m_1) placed at a distance L from the midpoint of the box.

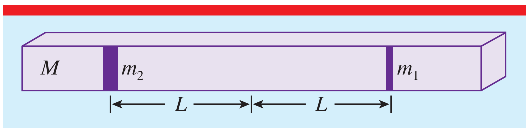


Figure 3. Black box with two unequal masses (m_1 and m_2) placed at equal distances L from the midpoint of the box.

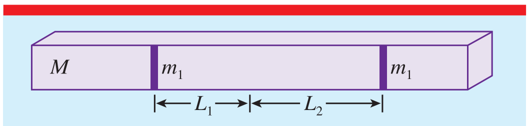


Figure 4. Black box with two equal masses (m_1) at two unequal distances (L_1 and L_2) from the midpoint of the box.

Figure 5. The black box.

Figure 1 describes different parts of an empty black box. This is essentially a long (~40 cm) narrow metallic box (breadth ~ 2.5 cm and depth ~ 2 cm), usually made from a U-shaped aluminum channel (sheets 1 and 2 make an aluminum channel). The upper surface is covered with a plastic sheet (sheet 3) that behaves like a lid. The box is sealed at the ends with two very light pieces of plastic sheet (sheet 4) of negligible mass, after the placement of the masses at known distances.

This box (figure 5) contains two masses of small size and regular shape fixed symmetrically on the axis of the box. The main objective of the experiment is to determine the unknown masses and their positions within the box. The mass of the empty box (M) normally needs to be supplied. Figures 2–4 show sketches of boxes that contain different combinations of masses and distances from the midpoint of the uniform empty box.

In this experiment, two unequal masses are placed at a distance L from the midpoint of the rectangular box (figure 3).

First the box is used like a beam of a balance and moments about some suitable points are taken using external known weights by balancing it over a knife edge (figure 6). This helps to develop some relationships involving the unknown masses and distances.

Two different relationships emerge.

(1) The geometric midpoint of the box is made the fulcrum. In this case, the weight of the empty box cannot contribute towards the moment about that point.

(2) Any other point is chosen as the fulcrum. Here the weight of the empty box does contribute to the moments.

The second type of operation involves the bifilar oscillation of the box about its centre of mass. This point is normally not the geometric midpoint after being internally unevenly loaded with unknown weights. Here we shall describe the principle of the experiment with two unequal masses m_1 and m_2 that have been placed at an equal distance L on both sides of the midpoint of the empty box. Here the mass of the empty box is also supplied, reducing the unknown quantities to m_1, m_2 and L.

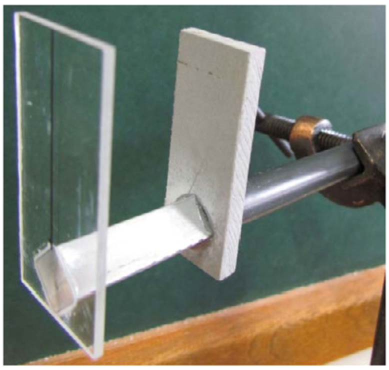


Figure 6. Knife edge with a marker on the front side.

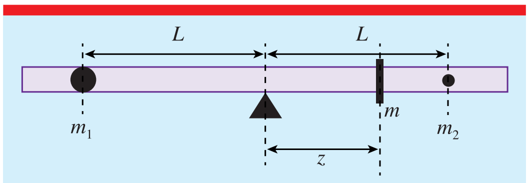


Figure 7. Moment of MBB about geometrical centre.

Experiment

Known quantities.

M—mass of the empty box = 115.0 g.

m—mass in the hanger + mass of the plastic holder (mass of the plastic holder = 1.0 g) (figure 8).

z—distance of mass m from the midpoint of the box.

Step 1: taking moments about the midpoint of the box by balancing it on the knife edge.

As per figure 7:

$$\begin{aligned} m_1L - m_2L - mz &= 0 \\ (m_1 - m_2)L - mz &= 0 \\ mz &= (m_1 - m_2)L \\ \therefore\ z &= \frac{1}{m}(m_1 - m_2)\,L. \end{aligned} \tag{1}$$

From the graph of z versus $\frac{1}{m}$ we obtain slope = $(m_1 - m_2)L$.

Observations. See table 1 and figure 9.

$$\text{Slope} = (m_1 - m_2)\,L = 449.28 \text{ g cm}.$$

Step 2: taking moments about a point between the unknown mass and the midpoint of the box by balancing it on the knife edge.

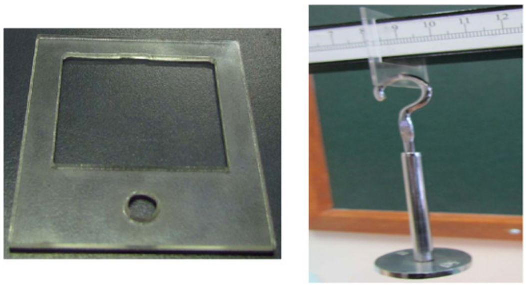


Figure 8. Hanger attached to the rectangular plastic holder.

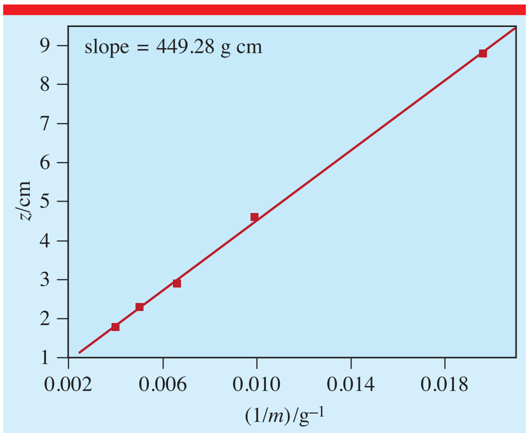


Figure 9. Graph of z versus $1/m$.

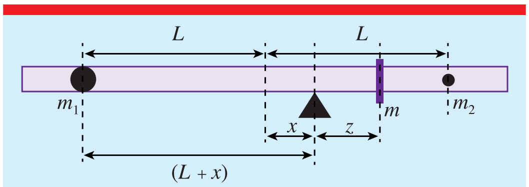


Figure 10. Moment about the centre of mass with known masses in the hanger.

From figure 10:

$$\begin{aligned} &m_1\,(L + x) - mz - m_2\,(L - x) + Mx = 0 \\ &\therefore\ (m_1 + m_2 + M)x = mz - (m_1 - m_2)L. \end{aligned}$$

If m is varied while keeping the position of the hanger fixed, then a graph of x versus mz can be plotted.

Table 1. Position of the geometrical centre of the black box, $q = 20.1$ cm.

Mass in the hanger (m_h) (g)	$m = m_h + 1.0$ (g)	$1/m$ (g^{-1})	Position of the hanger h (cm)	$z = h - q$ (cm)
50.0	51.0	0.0196	28.9	8.8
100.0	101.0	0.0099	24.7	4.6
150.0	151.0	0.0066	23	2.9
200.0	201.0	0.0050	22.4	2.3
250.0	251.0	0.0040	21.9	1.8

Table 2. Varying the mass in the hanger and keeping the position of the hanger fixed = 35.0 cm (CG—centre of gravity).

Mass in the hanger (m_h) (g)	$m = m_h + 1.0$ (g)	CG (cm)	$z = 35.0 - \mathrm{CG}$ (cm)	$x = \mathrm{CG} - 20.1$ (cm)	mz (g cm)
250.0	251.0	27.4	7.6	7.3	1907.6
200.0	201.0	26.4	8.6	6.3	1728.6
150.0	151.0	25.2	9.8	5.1	1479.8
100.0	101.0	23.6	11.4	3.5	1151.4
50.0	51.0	21.3	13.7	1.2	698.7

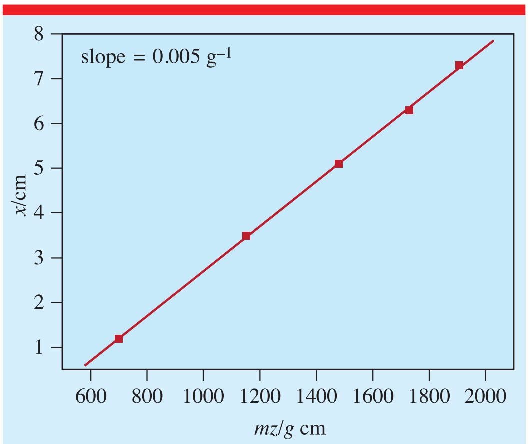


Figure 11. Graph of x versus mz.

In that case,

$$x = \left(\frac{1}{(m_1 + m_2 + M)}\right) mz - \frac{(m_1 - m_2)}{(m_1 + m_2 + M)} L \tag{2}$$

$$\text{Slope} = \frac{1}{(m_1 + m_2 + M)}.$$

Observations. See table 2 and figure 11.

$$\text{Slope} = \frac{1}{(m_1 + m_2 + M)} = 0.005\ \mathrm{g}^{-1}$$

$$(m_1 + m_2 + M) = 200.0\ \mathrm{g}.$$

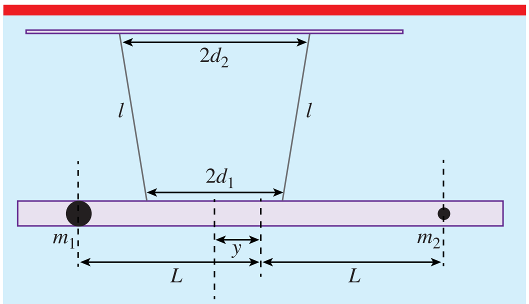
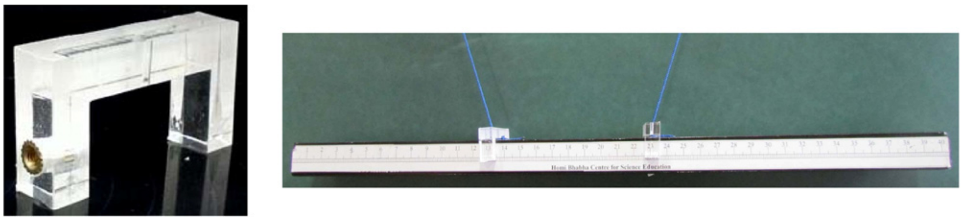


Figure 12. Mechanical black box suspended as a bifilar pendulum.

Step 3: suspending the MBB as a bifilar pendulum and studying torsional oscillations of the box.

When two strings are symmetrically tied on the two sides of the centre of mass of a body to allow it to have a torsional oscillation, then the suspension is referred to as bifilar suspension (figure 12). Here the MBB has been given a torsional oscillation about a vertical axis passing through its centre of mass.

(a) Two C-shaped plastic holders. Two C-shaped plastic holders (figure 13(a)) are used in step 3 to hold the black box in a bifilar pendulum configuration, as shown in figure 13(b). The holder can be clamped to the black box by tightening the screw with an Allen key.



(a) (b)

Figure 13. (a) C-shaped plastic holder. (b) Plane side of the holders on the scale side of the black box.

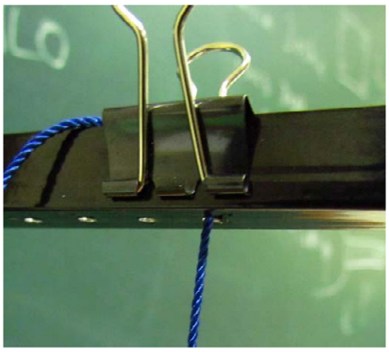


Figure 14. Binder clip used to hold the thread.

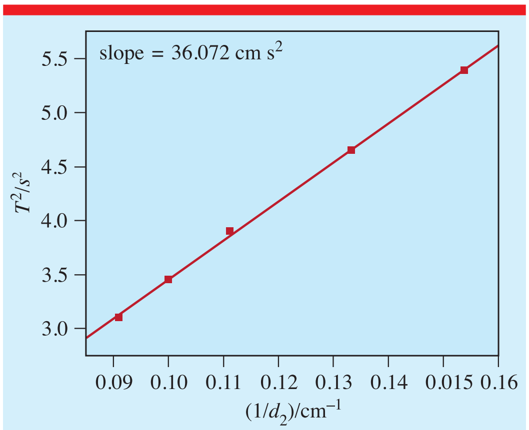


Figure 15. Graph of T^2 versus $1/d_2$.

(b) U-shaped channel with binder clips. The U-shaped channel has holes drilled at regular distances (1 cm apart). In step 3, threads can be inserted through these holes. Binder clips can be used to hold the thread in position, as shown in figure 14.

The time period of the torsional oscillations of the bifilar pendulum is given by [3]

$$T = 2\pi\sqrt{\frac{Il}{M'gd_1d_2}} \quad (3)$$

where $M' = M + m_1 + m_2$.

I—total moment of inertia of the box about an axis passing through its CG and perpendicular to its plane.

Observations. See table 3 and figure 15.

Table 3. $y = 2.0$ cm, $2d_1 = 12.0$ cm, $l = 33.0$ cm.

$2d_2$ (cm)	$1/d_2$ (cm^{-1})	T (s)	T^2 (s^2)
22.0	0.0909	1.761	3.101
20.0	0.1000	1.858	3.452
18.0	0.1111	1.975	3.901
15.0	0.1333	2.156	4.648
13.0	0.1538	2.321	5.387

Slope $= 36.072$ cm s^2

$$\therefore\ I = \frac{T^2 d_2 M' g d_1}{4\pi^2 l}$$

$$= \frac{(\text{Slope})\, M' g d_1}{4\pi^2 l}$$

$$= \frac{36.072 \times 200.0 \times 980 \times 6.0}{4\pi^2 \times 33.0}$$

$$= 32\,594.5 \text{ g cm}^2.$$

The moment of inertia, I, comprises three parts

$$I = I_1 + I_2 + I_3.$$

I_1 = moment of inertia of the empty box = 15 800 g cm^2.

(Note: for the calculation of I_1, the masses of the horizontal and vertical sections of the black box (figure 1) are separately provided.)

I_2 = moment of inertia of m_2 with respect to the centre of gravity, so $I_2 = m_2(L+y)^2$.

I_3 = moment of inertia of m_1 with respect to the centre of gravity, so $I_3 = m_1(L-y)^2$.

It needs to be mentioned in this connection that the diameters of unknown masses were much less than L. Hence the individual moments of inertia of the unknown masses are neglected.

$$I = I_1 + I_2 + I_3$$
$$I = I_1 + m_2(L+y)^2 + m_1(L-y)^2$$
$$32\,594.5 = 15\,800 + m_2(L^2 + 2Ly + y^2) + m_1(L^2 - 2Ly + y^2)$$
$$(m_1+m_2)L^2 - 2y[(m_1-m_2)L] + (m_1+m_2)y^2 - 16\,794.5 = 0$$
$$(M'-M)L^2 - 2y[(m_1-m_2)L] + (M'-M)y^2 - 16\,794.5 = 0$$
$$(200.0-115.0)L^2 - 2\times 2.0\times[449.28] + (200.0-115.0)\times 2.0^2 - 16\,794.5 = 0$$
$$(85.0)L^2 - 1797.12 + 340.0 - 16\,794.5 = 0$$
$$L^2 = 214.72\ \text{cm}^2 \qquad L = 14.653\ \text{cm}$$
$$(m_1-m_2)L = 449.28\ \text{g cm}$$
$$(m_1-m_2) = 30.66\ \text{g}, \qquad (m_1+m_2) = 85.0\ \text{g}$$
$$m_1 = 57.83\ \text{g} \quad \text{and} \quad m_2 = 27.17\ \text{g}$$

$$m_1 = 58 \pm 2\ \text{g}, \qquad m_2 = 27 \pm 2\ \text{g}$$
and $\qquad L = 15 \pm 2$ cm.

Actual values:
$$m_1 = 58.0\ \text{g}, \qquad m_2 = 29.0\ \text{g}$$
and $\qquad L = 15.0$ cm.

Conclusion

The mechanical black box (MBB) problem is interesting because it can be used in several variations. We are in a position to determine three unknowns by framing three independent equations. So the problem of the MBB may be framed in several distinct ways by changing unknown quantities while always keeping their number to three. However, in the process we need to provide some additional information depending on the problem. Here are a few examples of different problems with MBBs where the number of unknowns may be different from that discussed here.

(a) m_1 (one unknown mass), L (location of the unknown mass with respect to the midpoint of the empty box), M (mass of the empty box) (as shown in figure 2).

(b) m_1 (one unknown mass), m_2 (second unknown mass), L (equal distance from the midpoint of the box), M (mass of the empty box is supplied) (as shown in figure 3).

(c) m_1 (one unknown mass), m_2 (second unknown mass), M (mass of the empty box). (Two unknown masses are kept on the two sides of the midpoint of the box and the distance(s) L_1 and L_2 are supplied.)

(d) m_1 (mass of each equal unknown mass), L_1 (distance of one mass from the midpoint), L_2 (distance of other equal mass from the midpoint), M (mass of the empty box is supplied) (as shown in figure 4).

One can actually handle the MBB with only two unknowns (say a mass and its location inside the box) just by taking moments about the midpoint of the box and about any other suitable point. So, by using equations (1) and (2) with the value of the mass of the empty box supplied, unknowns can be determined. That would actually be the simplest type of MBB, where the bifilar oscillation is not necessary, meaning that the set-up could act as an introductory MBB in a middle school class where the students have just learnt about the moments of forces and turning effects, but have not yet been introduced to oscillations.

Another important aspect of this exercise lies in the fact that the students can actually fabricate the MBB with easily available inexpensive materials and can perform the experiment by themselves. In fact, it is easy to make multiple sets and give them to a number of students who can then perform the experiment simultaneously in a laboratory situation.

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