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Alternative conceptions in Galilean relativity: inertial and non-inertial observers

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This concluding part of a study on Galilean relativity focuses on students' notions with regard to the inertial and non-inertial character of frames of reference. (See Panse *et al*. 1994, Ramadas *et al*. 1996). The results show that students: adopt kinematic criteria for deciding the inertial or non-inertial character of frames; consider this character to be a 'relative' property of two frames rather than an intrinsic property of a given frame; and equate pseudo-forces to 'imaginary' forces. Centrifugal force is associated with rotating objects rather than with rotating frames; the latter are localized by the finite extension of their associated objects. Anthropomorphic criteria are invoked to judge the existence of centrifugal force, which is regarded as a reaction (in the sense of Newton's third law) to the centripetal force on a rotating object.

Introduction

The first two parts of this study dealt with students' notions on 'frames of reference' (Panse *et al*. 1994) and Galilean transformations (Ramadas *et al*. in press). This concluding part probes students' conceptions regarding inertial and non-inertial character of frames of reference and pseudo-forces, particularly the centrifugal force. Alternative frameworks on the centrifugal force at a more elementary level have been the subject of some earlier studies (e.g. Gardner 1984).

Qualitative phenomenology of alternative conceptions

A pilot version of a free-response test was administered to 17 college students. After certain modifications, the text (Appendix A) was administered to a sample of 29 physics undergraduates from a Bombay college. Four students were selected for clinical interviews. Qualitative analysis of the data led to the following phenomenology of alternative conceptions (AC).

AC (I) A frame is inertial if you are with it; non-inertial if, when looked at from 'outside', it is rotating/accelerating

That Newton's first law of dynamics *defines* an inertial frame is frequently stressed in physics teaching. What is perhaps rarely emphasized is the negative assertion: there is no *a priori* kinematic criterion for deciding if a frame is inertial or non-inertial. Lack of appreciation of this fundamental point is germane to many alternative conceptions examined in this paper. Thus when called upon to comment on the inertial or non-inertial nature of a frame of reference in particular situations, students readily invent some simplistic kinematic criteria for the purpose.

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The recurrent view that emerges is that a frame is inertial if it is 'stationary' or if it may be considered stationary for the purpose at hand. By this criterion, regardless of what frame of reference one considers, if you perform experiments 'on' it or 'near it', the motion of the frame is irrelevant, and 'hence' the frame is inertial. A variant of the same view in the context of Q.7 is that the earth's velocity is large and the relative velocities of the bodies on it are much smaller; hence the earth's frame is inertial for terrestrial experiments. Yet another variant relates the inertial property of a frame to the relative 'masses' of the bodies and the frame: the earth's frame is inertial for terrestrial experiments since the earth is far more massive than other bodies on it. (See also AC (II) below.)

Why is the earth then a non-inertial frame for astronomical observations? Well, as long as you are on it, you do not see its motion; so it is inertial. If you want to see the motion of the frame, you must go 'outside' the frame to judge its non-inertial character. From 'outside' the earth's frame, we know it is revolving and rotating. 'Hence' the earth is non-inertial for astronomical observations. These lines paraphrase a rather common view that emerged from students' responses. The view embodies both AC (I) and AC (VII) regarding localization of frames.

AC (II) Some rotations are real, some apparent

As it happens, AC (II) is a correct conception in physics: unlike uniform motion which is relative, rotation and acceleration are absolute, i.e. ascertainable from measurements relative to a frame. We call it an alternative conception, nonetheless, because students' responses suggest that their 'correct' conception was an extension to rotational motion of the idea of the 'real-apparentness' of (translational) motion diagnosed earlier (Panse *et al.* 1994).

Markers for AC (II) are the views (in the context of Q.4) that the man outside the merry-go-round is actually at rest, the centrifugal force on the rotating man (in the merry-go-round's frame) is 'apparent' and the claim that the man is rotating with opposite ω (angular velocity) is wrong. In Q.3, AC (II) is 'justified' in terms of the relative masses of the sun and the earth, akin to the way the earth's inertial nature for terrestrial experiment is 'explained':

> 'It is the earth that rotates round the sun, because mass of the earth is very small compared to the mass of the sun; it is quite obvious that a particle with small mass can rotate around a heavy body, not a heavy particle around a small particle.'

The conflict between real-apparentness of rotation and the principle of relativity is sometimes resolved by invoking a third (neutral) observer:

> 'If we become a "third person" and go away from the earth, and if we remain at rest, then we will clearly be able to say that the sun is also at rest and the object which is moving is the earth.'

One cannot fail to draw a close parallel between this student's 'third person' and the Newtonian conception of a frame of absolute rest (ether).

AC (III) All motion is relative (Ultra-relativism)

One unexpected outcome of this study is the realization that a student well aware of the principle of relativity can carry it to an extreme: all frame of reference are on the same footing. AC (III) is the opposite of AC (II). Relative to the earth, the sun moves across the sky. Relative to the sun, the earth moves, and there is nothing to prefer one description to the other. When asked in an interview why the sun is usually considered to be a better frame of reference, the student said that this was because the sun was at the centre of our planetary system. What if the sun had just one planet? Would the sun's frame be still preferable? 'It is just a system of two bodies rotating about each other', replied the student, 'Why should the frame of reference of one body be more privileged than the other?'

AC (III) is a standard conception of physics in that it is possible to formulate laws of physics which manifestly show equivalence of all frames of reference (general theory of relativity). But that is not the sense in which it is held by students. In the coarse sense of denying absolute acceleration/rotation and banishing the distinction between inertial and non-inertial observers, ultra-relativism can rightly be called an alternative conception, at variance with physics.

AC (IV) Inertial or non-inertial character is a relative property between frames of reference

If a frame of reference S is inertial, Galilean transformations (more generally, Lorentz transformations) ensure that any other frame moving uniformly relative to S is also inertial. In the teaching of relativity, it is very customary to consider 'two inertial frames S and S′ in uniform relative motion'.

AC (IV) is probably a result of the overuse of the underlined statement, combined with AC (III). This origin of AC (IV) is merely our conjecture; but its existence is a fact: 'The same frame of reference, say S_1, may be moving with uniform velocity with respect to another frame of reference S_2, but it may *not* be moving with uniform velocity with respect to a third frame of reference S_3. Thus S_1 is inertial with respect to S_2 but non-inertial with respect to S_3.' This binary attribute was also seen to satisfy reciprocity: the student held the view (Q.4) that for the man outside, the child is rotating and is a non-inertial observer; for the child the man is rotating, hence the man is non-inertial. This same student also held AC (III) firmly.

AC (V) Centrifugal force acts on rotating objects

A common view among students is that centrifugal force acts on every rotating body. Thus a revolving stone or a child in a merry-go-round are acted upon by centrifugal force, regardless of the frame of reference. Objects which are steady (like the man outside the merry-go-round) do not have centrifugal force on them. Clearly, the view negates the correct conception that centrifugal force – a pseudoforce – is to be invoked only in a non-inertial frame of reference.

One response indicated that a natural ground for AC (V) may be that centrifugal force is perceived as a reaction (in the sense of Newton's third law) to the centripetal force required to keep a body in circular motion.

The origin of AC (V) probably lies in the 'experience' of centrifugal force (in a merry-go-round, in a vehicle negotiating a bend, etc.), and also in wrong instruction. Informal observations indicate that AC (V) is passed on to students in connection with the explanation of the 'equilibrium' of a body in uniform circular motion. The explanation is right only in the rotating body's frame of reference and not in the laboratory frame – a fact that is rarely emphasized.

AC (*VI*) *Judging forces by anthropomorphic criteria*

A common anthropomorphic criterion is to equate force with its 'feeling' or physiological 'experience'. If you are on a merry-go-round you 'feel' being pushed out – 'hence' there is a centrifugal force on you. By this criterion, there is a centrifugal force on the child but not on the man – the child feels the centrifugal force, the man does not.

AC (VI) is a natural but simplistic way of thinking; it persists since students are rarely told the important point that the 'feeling' of a force arises only when our body (a non-rigid flexible system) develops internal stresses to prevent any relative motion between its various parts. It is these stresses, not the net external force, that give rise to the 'feeling' of force.

A less common anthropomorphic criterion for centrifugal force is its related visual effects. You can see the widening between the bars of a merry-go-round as its starts to rotate, 'hence' there is a centrifugal force on the child. The man on the ground sees the rotating child pressed against the seat, 'hence' there is a centrifugal force on the child in the man's frame.

AC (*VII*) *Localizing rotating frames by the extension of the associated objects*

AC (VII) is part of the general tendency of students to localize a frame of reference (Panse *et al.* 1994). In this view, the rotating frame of reference of a turntable goes up to the edge of the turntable. 'Beyond' that is the inertial frame of the ground.

Though students may disown this conception if put in an exaggerated form, it still shows up implicitly even among well-trained students. One response to Q.4 contained the categorical (and correct) statement that 'centrifugal force can be considered only for non-inertial frames of reference', which showed a clear absence of AC (V). Yet in the same response were phrases such as 'the child is in the rotating frame of reference'; 'the man on the ground is in the inertial frame' – clear pointers to the conception of localization. A corollary of AC (VII) shows up naturally in some responses: the man on the ground experiences no centrifugal force since he is 'outside' the non-inertial frame of the merry-go-round.

AC (*VIII*) *Pseudoforces are 'apparent' or imaginary forces*

AC (VIII) says that centrifugal force actually does not exist; it is merely invoked to explain rotational phenomena. This is a correct conception if, by that, one means that the centrifugal force arises not from any material or physical agency, but from the intrinsic nature of the frame of reference. But that is not what is intended by students when they use words like 'apparent' or 'imaginary' to qualify centrifugal force. A most telling evidence of AC (VIII) appeared in an interview when a student was reluctant to cancel a 'real' force (like tension) with a pseudo-force like a centrifugal force. 'If the two forces cancel, why should one feel the centrifugal force at all?' This response indicates AC (VI) also.

AC (VIII) probably arises partly from the linguistic connotation of the word 'pseudo'. It would be interesting to test if, by using terms like 'inertial forces' or 'frame-dependent forces', this conception could be remedied.

One observation that emerged from Q.2 and Q.5 was that for situations involving translational motion, students prefer inertia-based explanations, while for rotational

Table 1. AC (I): using kinematic criteria.

AC (I)	*Aspects of AC (I)*	*I*	*Prevalence %*	*Certainty Index %*
A	Non-inertial character of a frame has to be determined from 'outside' the frame.	4e	73	34
B	A frame is inertial if you perform experiments 'on it' or 'near it' (in which case you can consider it to be stationary).	4a	69	53
C	The earth being very massive remains stationary and hence its frame is inertial	4d	51	33
D	Earth's frame is inertial since its velocity is large compared to that of objects on it (*and therefore can be taken to be a constant*).	4c	47	31

motion, they invoke pseudo-forces. This tendency is probably rooted in instruction, and was not probed further.

Force-option test results

The forced-option test (Appendix B) was administered to a sample of 77 senior physics undergraduates in Bombay. As in the earlier parts of the study (Panse *et al.* 1994, Ramadas *et al.* in press), the Pearson correlation between prevalence and certainty index was strong (0·62, $\sigma = 0{\cdot}001$). Most of the items in this part, however, showed a low certainty index, perhaps reflecting the higher technical level of the subject.

AC (I): Using kinematic criteria for judging the inertial or non-inertial character of a frame of reference.

The four different aspects of AC (I) in I.4 appear in Table 1. The consistency between the two aspects A and B of AC (I) was not particularly good ($\chi^2 = 2{\cdot}89$, $\sigma = 0{\cdot}09$). Aspects C and D are two different 'justifications' for the inertial character of the earth's frame. Aspect D is particularly intriguing. The italicized statement in Table 1 for AC (I)D is our conjecture about the underlying reason for the conception. A fairly large proportion of students (58%) agreed with I.4b, the correct response. Clearly, students' learnt ideas coexist with the loosely consistent set of kinematic criteria they possess for ascertaining the inertial or non-inertial character of a frame.

AC (II): Some rotations are real, some apparent.

Results of AC (II) are shown in Table 2. Note that since I.1e and I.3f are comound statements, responses to them are influenced by aspects of AC (II) and AC (V). Similarly, responses to I.1i and I.3k are influenced by AC (II) and AC (VIII). There

Table 2. AC (II): **some rotations are real, some apparent.**

AC (II)	*Aspects of AC (II)*	*I*	*Prevalence %*	*Certainty Index %*
A	Some objects are 'in fact' stationary (hence no cf on them).	1e 3f	61 53	51 44
B	On objects which are apparently rotating, cf is 'apparent'.	1i 3k	69 70	51 41
C	Absoluteness of rotation is related to relative masses.	6a	51	36

is high consistency between the responses to I.1e and I.3f ($\chi^2 = 17{\cdot}25$, $\sigma = 0{\cdot}0000$), and also between those to I.1i and I.3k ($\chi^2 = 9{\cdot}41$, $\sigma = 0{\cdot}002$). The difference between between responses to I.1e and I.3f and also between I.1i and I.3k was not significant. It appears that anthropomorphic contexts do not materially affect AC (II).

AC (III): All motion is relative. (Ultra-relativism)

AC (III) was tested in I.6b directly, and somewhat indirectly in I.1g and I.3i (Table 3). Here, again the responses to I.1g and I3i are influenced by aspects of AC (V). The consistency between these responses was found to be low ($\chi^2 = 0{\cdot}32$, $\sigma = 0{\cdot}57$).

Table 3. AC (III): **ultra-relativism.**

AC (III)	*Aspects of AC (III)*	*I*	*Prevalence %*	*Certainty Index %*
A	All motion is relative. It is wrong to say that either the sun or the earth is 'actually' stationary.	6b	55	52
B	Rotation is relative. Objects rotating relative to a frame experience cf, stationary objects do not.	1g 3i	44 38	56 59

Table 4. AC (IV): **inertial/non-inertial nature is relative.**

AC (IV)	*Aspects of AC (IV)*	*I*	*Prevalence %*	*Certainty Index %*
A	Uniform relative motion implies relative 'inertialness'	5c	66	47
B	Relative acceleration implies relative 'non-inertial-ness'.	5b	60	41

AC (IV): Inertial or non-inertial character is a relative property between frames of reference.

Results for AC (IV) are summarized in Table 4. The consistency between aspects A and B of AC (IV) was found to be low ($\chi^2 = 2{\cdot}1$, $\sigma = 0{\cdot}15$). Further, 73% of the students also accepted the correct alternative. It is clear that the inconsistency between the standard conception and AC (IV) is not appreciated by most students.

AC (V): Centrifugal force acts on rotating objects

Six different aspects of AC (V) were tested (Table 5). Within each group of questions corresponding to a given aspect, there was a high degree of internal consistency. High prevalence and high certainty of certain aspects of AC (V), and low prevalence and low certainty of the standard concepts, is an important result of this work. The alternative framework emerging from aspects A and B of AC (II) and aspects A, D and E of AC (V) may be described thus: Centrifugal force acts on rotating objects; rotation may be real or apparent; centrifugal force on truly rotating objects is real whereas that on an apparently rotating object is apparent.

Table 5. AC (V): rotating objects experience centrifugal force.

AC (V)	*Aspects of AC (V)*	*I*	*Prevalence %*	*Certainty Index %*
A	In lab/ground frame, rotating objects have cf, steady objects do not.	1a 2a 3a	75 83 86	78 69 79
B	Rotating objects have cf in their own frame.	1b 3b	57 64	45 55
C	No matter which frame, rotating objects have cf, not steady objects.	1c 3c	49 48	50 32
D	No cf on bodies which are 'in fact' stationary.	1e 3f	61 53	51 44
E	cf on objects not 'truly rotating' is apparent	1i 3k	69 70	51 41
F	cf and centripetal force are action-reaction pair.	2b	70	63

Table 6. AC (VI): judging forces and anthropomorphic criteria.

	Aspects of AC (VI)	*I*	*Prevalence %*	*Certainty Index %*
AC (VI)	If an object 'feels' a centrifugal force, there is a cf, otherwise not.	3d 3g	61 52	66 35

Table 7. AC (VII): localizing frames by associated objects.

	Aspects of AC (VII)	*I*	*Prevalence %*	*Certainty Index %*
AC (VII)	cf acts on an object 'inside' a frame; not on an object 'outside'.	1d 3e	64 58	63 49

Table 8. AC (VIII): pseudo-forces are not 'real' forces.

	Aspects of AC (VIII)	*I*	*Prevalence %*	*Certainty Index %*
AC (VIII)	cf, being fictitious force, cannot balance a 'real' force	2d	21	13

AC (VI): Judging forces by anthropomorphic criteria

Results for AC (VI) are summarized in Table 6. Students' responses were not very consistent and this finding is difficult to interpret. Despite much effort, questions in the test are sometimes overtly suggestive and this leads to guarded responses by the students. We suspect anthropomorphism is more prevalent than our data indicate; a more thorough investigation is needed.

AC (VII): Localizing rotating frames by the extension of the associated objects

Results for AC (VII) are given in Table 7. The consistency between the responses to the two items was fair ($\chi^2 = 5{\cdot}59$, $\sigma = 0{\cdot}02$).

AC (VIII): Pseudo-forces are 'apparent' or imaginary forces

Results for AC (VIII) are summarized in Table 8. AC (VIII) by itself is not very prevalent. However, when it combines with real-apparentness of rotation [AC (II)] it gives rise to aspect E of AC (V) which has high prevalence as noted earlier.

Conclusions

A physicist decides the inertial nature of a frame of reference by a purely empirical criterion: the validity of the first law of motion. (If a no-force situation required by the first law is impractical, the inertial nature of a frame is tested by seeing if the second law works without having to invoke pseudo-forces). This criterion is *intrinsic*, i.e. it needs no other frame for comparison. However, if some frame of reference, say S_1, has been so tested to be inertial, the inertial or non-inertial character of any other frame (S_2) can be ascertained by a relative kinematic criterion: S_2 is inertial (non-inertial) if it moves uniformly (non-uniformly) with respect to S_1.

We have seen that students' conceptions respect neither the empirical nor the intrinsic nature of the criteria. Most students adopt simplistic *a priori* criteria: if you are 'with' the frame, it is inertial; if from 'outside' you see it accelerating or rotating, it non-inertial. Further, they violate the intrinsic nature of the property by viewing it as a relative reciprocal property between two frames. This last view combined with the (learnt) principle of relativity can sometimes lead to ultra-relativism, where all frames of reference are considered equivalent (in naïve sense) and the distinction between inertial and non-inertial frames is banished.

Students' *a priori* kinematic criteria in a way implicitly subsume their other important alternative frameworks based on localization of frames of reference and anthropomorphic reasoning. These latter conceptions, however, show up independently and markedly in connection with situations involving rotating frames and centrifugal forces. When these frameworks combine with students' value judgements on real and apparent motion, they result in the variety of confusions and inconsistencies students display in dealing with simple problems on inertial and non-inertial frames.

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Appendix A

The free response test

Q.1. Image yourself sitting steady in a rotating merry-go-round. In which direction do you feel yourself being pushed? Why? Then what prevents our motion in that direction?

Q.2. Imagine yourself sitting in an accelerating bus, facing forwards. In which direction do your feel youself being pushed? Why? Then what prevents your motion in that direction?

Q.3. Motion, it is said, is relative. For example, on the earth's frame of reference, the sun moves across the sky each day from east to west; while in the sun's frame of reference the earth rotates about its axis. Yet we insist that the motion of the sun around the earth is apparent, and it is the earth that actually rotates. What is the basis for this preference? (Please ignore the presence of other planets in the solar system.)

Q.4. A child is sitting in a rotating merry-go-round and a man in standing on the ground. In the ground's frame of reference, is there a centrifugal force on the child? In the child's frame of reference, is there a centrifugal force on the man? Explain.

Q.5. A train is moving uniformly. A ball rests on the floor of the train. If the train slows down suddenly, describe and explain the motion of the ball both from the point of view of a train observer and a ground observer. Ignore friction.

Q.6. A stone is tied to a string and rotated uniformly in a horizontal circle. Is the following statement true? 'The tension in the string and the centrifugal force on the stone are equal and opposite, and therefore keep the stone in equilibrium.' Explain your answer.

Q.7. It is said that the earth is an inertial frame for most terrestrial experiments, but a non-inertial frame for astronomical observations. How can the same frame of reference be inertial for one purpose and non-inertial for another? Explain.

Appendix B

The forced option test and results

In the various problem situations below, a series of statements are given. Some of these statements are followed by the options 'a, b, c, d'. Please select one of the four options by circling it, using the following key

a: The statement is definitely true.
b: Not sure, but the statement might possibly be true.
c: Not sure, but the statement appears to be wrong.
d: The statement is definitely untrue (or) it does not make sense.

Consider the statements only in the given sequence. Do not go back to any question that you have already read.

[The results are summarized in terms of the percentages of students who agreed with, disagreed with, or gave no response to the given statement. The correct response are underlined.]

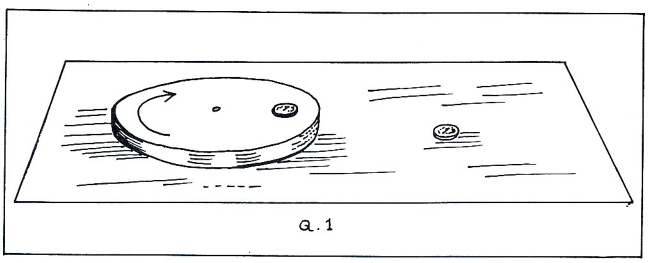

Figure 1.

		Agree %	*Disagree* %	*No response* %
I.1	A turntable kept on a platform rotates clockwise with a constant angular speed. Coin 1 rests on the rotating turntable, while coin 2 rests on the platform. (see Figure 1)			
1a.	In the lab frame, there is a centrifugal force on the rotating coin 1 but not on the stationary coin 2.	75	<u>22</u>	3
1b.	In the rotating turntable's frame, there is a centrifugal force on coin 1 but not on coin 2.	57	<u>40</u>	3

		Agree %	Disagree %	No response %
1c.	No matter which frame, there is a centrifugal force on coin 1 but not on coin 2.	49	48	3
1d.	There is no centrifugal force on coin 2 in either frame, because coin 2 is outside the turntable.	64	35	1
1e.	There is no centrifugal force on coin 2 in either frame, because coin 2 is in fact stationary.	61	36	3
1f.	In the lab frame there is no centrifugal force on either coin 1 or coin 2.	17	78	5
1g.	In the rotating turntable's frame coin 2, which appears to rotate (anticlockwise) has a centrifugal force, but coin 1, which is stationary with respect to the frame, does not have a centrifugal force.	44	53	3
1h.	In the rotating turntable's frame, both coins 1 and 2 have centrifugal force.	36	61	3
1i.	The centrifugal force on coin 2 in the rotating frame is only apparent (i.e. it really does not exist), because coin 2 is actually at rest.	69	26	5
I.2	A stone is tied to a string and rotated uniformly in a horizontal circle.			
2a.	In the lab frame, there is a centrifugal force on the rotating stone.	83	17	0
2b.	The centripetal force (due to tension in the string) and the centrifugal force on the stone are equal and opposite by Newton's III law.	70	30	0
2c.	In the rotating stone's frame, the centrifugal force on the stone is balanced by the force due to tension.	79	17	4
2d.	Centrifugal force cannot balance tension because centrifugal force is a fictitious force; it cannot balance a real force like tension.	21	75	4