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Alternative conceptions in Galilean relativity: frames of reference

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In this paper we investigate how students handle the abstract conceptual tool, namely, 'frames of reference', that is needed for the formulation and exploitation of the principle of relativity in physics. The methodology of probing into students' constructs is partly holistic and partly analytical. First, through a free-response test and selected clinical interviews, we obtain holistic impressions of students' notions and assign them to a number of interpretative categories of responses. These are then grouped under a smaller number of superordinate categories of students' alternative conceptions. Guided by this phenomenology, a diagnostic forced-option test is designed using a variety of situational contexts. The test is administered on a sample of 111 physics undergraduates drawn from different Bombay colleges. Students' responses are analysed to see the extent to which the alternative conceptions are held, and the consistency or otherwise between their various aspects. Our data show unmistakably that the more prevalent alternative conceptions are also the ones which are held with greater conviction. The analysis indicates that students implicitly associate 'frames of reference' with concrete objects, localized and bounded by the latter's extension; regard particular phenomena as 'belonging' to particular frames; allow value judgement on 'real' and 'apparent'[ness] of motion to co-exist with their (learnt) knowledge about relativity of motion; and equate physical description to anthropomorphic viewing.

Introduction

Over the last more than a decade, investigation of students' domain-specific notions has been an active area of science educational research. This research is in large measure inspired by a constructivist viewpoint (originally due to Piaget), which holds that children are active constructors of their knowledge, and not empty vessels into which knowledge can be poured at will. Later empirical and theoretical studies (Siegler 1978, Forman and Pufall 1988) have attested to the domain-specific nature of cognition, thus parting with Piaget's version of constructivism involving abstract hierarchical cognitive stages. Statements on the constructivist epistemology in science learning can be found in Pope and Gilbert (1983), Driver and Oldham (1985), and Carey (1985). Pioneering empirical research has been done on children's alternative conceptions in specific domains of science by groups in the UK, Australia, New Zealand and the USA.

Most of this research, however, is concerned with elementary science. For example, some topics in elementary physics which have been extensively researched include: mechanics (McDermott 1984), geometrical optics (Ramadas and Driver 1989) and heat (Erickson 1979) (for a review, see Driver *et al.* 1985). However, there has been some work on somewhat more advanced topics also (Peters 1982, Lawson and McDermott 1987, Rozier and Viennot 1991). While most of this research has been descriptive, the literature on problem solving shows an analytic approach, usually in the information processing paradigm (Reif 1987, diSessa, 1988). In general, the investigations have revealed that students' conceptions have a marked degree of universality that cuts across different cultures, that they have a measure of internal consistency and, what is more important from the point of view of pedagogy, the alternative conceptions are fairly robust and resistant to formal training.

The purpose of the present investigation is to carry this work further to conceptually more complex domains of physics. Some care is, however, needed in the choice of the particular domain for this kind of inquiry. The domain should entail conceptual complexity and sophistication, and yet it should not be clouded by highly technical detail. The kinematics of relativity is a topic that suggests itself naturally as one suitable domain for this kind of work. We choose Galilean relativity as the specific domain in which to work and, in particular, take up the study of students' notions on 'frames of reference'.

Diagnostic methodology: free-response and forced-option tests

The standard method of investigation in studies of this kind consists in devising some relevant problem tasks for students, and carrying out careful clinical interviews. Students' written and oral responses are then subject to detailed qualitative analysis with a view to obtaining insights on dominant frameworks of their thinking. Our methodology is much the same; however, in order to handle students' notions in a more complex domain, we have extended it to include both analytical as well as holistic features, as explained below.

In the first exploratory phase of the study, a one-semester course on relativity was taught to a group of about 20 undergraduate students drawn from different colleges in Bombay. A multiple-teacher instructional strategy was adopted, and students' responses were informally observed. In this manner we got some tentative ideas about students' recurrent conceptual barriers in relativity.

These ideas formed the basis of a free-response test that was administered to another group of students. The sample consisted of 50 physics undergraduates in a Bombay college who had completed an introductory course in relativity. The test presented a number of different situations involving the notion of frames of reference. Students were asked to articulate freely the reasons for their responses. Each query had an expected correct response; students' responses were, however, not judged against this correct response, but were analysed independently. Appendix A gives the free-response test in detail.

To make sense of the large amount of qualitative data of students' responses, a holistic strategy was adopted. Each answer paper was read through and, instead of a question-wise evaiuation of the student's responses, we recorded some 'pointer' statements in the paper which seemed to paraphrase best our integrated impressions of the students' notions on frames of reference. These statements were then examined for key notions which emerged from them either implicitly or explicitly. In this way we coded all the answer-scripts in terms of a number of interpretative categories of responses. After this tentative categorization, clinical interviews were held with three of the students who had given the free-response test. These provided important feedback on the analysis, and gave additional clues to students' thinking.

The interpretative categories were then scrutinized for the existence of any natural groupings, or superordinate categories. Seven such groupings were identified. These represent broadly the dominant alternative conceptions about frames of reference, which appear to underlie the misconceptions and barriers that students reveal in particular problem tasks.

The free-response test analysis gave us a phenomenology of students' alternative conceptions, but it gave no indication of how widely these conceptions were held, how consistently they were used, and how sensitive they were to the context. To investigate these aspects, the second phase of the work consisted of designing a forced-option test, guided by the phenomenology arrived at earlier. The test contained a variety of diagnostic problem situations. Each problem situation had to do with some aspect of one or more of the seven categories of alternative conceptions. An interpretative key was developed relating students' choice of an alternative in each question to one of the alternative conceptions. After an initial pilot trial, the test was administered over a sample of 111 physics undergraduates. Using the interpretative key for students' responses, we finally obtained quantitative ideas on the prevalence of alternative conceptions among students, their different aspects and relative strengths in particular situations, and also the consistency with which these notions were used.

Qualitative phenomenology of alternative conceptions

In what follows we give the seven broad categories of alternative conceptions on frames of reference which were diagnosed through the free-response test analysis. Under each category, some sample responses and key notions are described, reinforced occasionally by our impressions obtained through the clinical interviews. Lastly, for each alternative conception, we note the specific forced-option questions we have designed for the quantitative analysis taken up in the next section.

Alternative conception (I) [AC(I)]: Treating frames of reference as concrete objects

Students had a tendency to talk about frames as if they were concrete objects, physically fixed to bodies. One student, for example, suggested that a ship and its frame of reference both suffer friction in water. Another student associated 'frames' with still and moving frames of pictures. Imagining frames as concrete objects was sometimes reflected in students' use of the word 'fixed' or 'not mobile': 'a frame of reference [is] a system of particles which do not have relative motion with respect to each other'. It seems, however, that at least in college students this conception is mostly held implicitly, and if students were to be probed on this point through direct (and suggestible) questions, their guarded responses would negate it. For instance, in the interview when a student was asked if the frame of reference of a plane would catch fire in an explosion, she laughed away the absurdity of the suggestion, even though she seemed implicitly to hold the notion of concreteness of frames in less direct situations.

The forced-option test investigated this conception in four different contexts: collision of airplanes (Q.1); a boat tossing in muddy water (Q.2), two trolleys pushed apart by the action of a spring (Q.3), and the sun–earth–moon system (Q.4).

Alternative conception (II) [AC(II)]: Localizing frames by the physical extensions of the objects they are 'fixed' to

In students' way of thinking, a frame of reference is defined not only by the characteristics of motion of the 'associated' body but also by the body's other parameters. A corollary of this idea is the commonly held notion that to each object there is associated a frame of reference. This was strongly indicated by students' insistence that frames translated with respect to one another without any relative motion were distinct frames. Another indication of AC(II) was students' frequent endorsement of the following statement: 'the vendor is walking in the [platform's] frame and the child is running in the [train's] frame'. A related response was that a ball could be thrown to 'go outside' a frame of reference.

The interviews reinforced these indicators. One student claimed that for a frame of reference fixed to a car, the body of the car was a limit; outside that limit might exist another frame. A most telling evidence of AC(II) in the interview was a student's associating a frame of reference with some kind of space in which the observer was sitting; the finiteness of that space was, of course, implicit in this view.

Two separate situations were designed in the forced-option test to diagnose AC(II). Q.4 seeks to probe AC(II) somewhat directly by offering bodies differing greatly in extension (sun, earth and moon), while Q.5 probes other manifestations of the same view, namely finiteness of the axes and 'boundaries' of frames.

Alternative conception (III) [AC(III)]: Treating small bodies located on a larger body as 'part of the frame' of the larger body

This interesting conception suggested itself from the response that, for a man walking on the deck of a ship, the ship would be at rest. Here it appeared that the student conceived of a space in the 'interior' of a frame in which relative motion was thought to be irrelevant. This conception is denoted by AC(IIIa) if it holds only for small velocities (with respect to the larger frame) and AC(IIIb) if it holds for arbitrary motion. These are investigated respectively in Q.7e and Q.7f of the forced-option test. We probe whether students change their view when the motion of a man on the deck is replaced by, say, motion of a bullet fired from the deck.

Alternative conception (IV) [AC(IV)]: Associating particular phenomena with particular frames

The absence of clear delineation between phenomena and frames of reference showed up in a number of ways. Two phrases which recurred in students' responses were: 'phenomenon (X) takes place in frame (S)'; 'the motion takes place relative to frame (S)'. On the face of it the second phrase hardly seems exceptionable. However, more careful reading of their overall responses to Q.2 of the free-response test suggested that students seemed to regard phenomena as 'taking place' relative to one frame and not to another frame. For students, particular phenomena belonged to particular frames, the latter being either the frame that the phenomenon 'took place in' (derived from AC(I) and AC(II) above) or the frame that the phenomenon was viewed from.

The alternative conception IV, however, emerged from the free-response test much less sharply than the other categories of conceptions because it was clouded by a number of ancillary confusions, which seemed to originate partly from the students' inadequate familiarity with the technical use of the term 'phenomenon' in physics. For example, one student equated two descriptions of the same phenomenon with two separate phenomena. The interview brought out yet another kind of confusion: motion of a ball thrown out of a moving train is a phenomenon neither for the platform's frame nor for the train's frame – it is a phenomenon for a third frame, namely, the frame of reference of the ball. Since no clear and significant underlying conception emerged from these responses, the forced-option test limited itself to diagnosing just one aspect: the 'belonging' of particular phenomena to particular frames of reference. Two contexts were employed: that of the 'closed frame' of a railway compartment (Q.6f and Q.6g) and of the 'open frame' of the deck of a ship (Q.7b to Q.7d).

Alternative conception (V) [AC(V)]: Real and apparent-ness of motions

The terms 'real (or actual) motion' and 'apparent motion' occurred frequently in students' free responses. This usage is perhaps related to the tendency to have a preferred convenient frame for describing a given phenomenon, which in itself is justifiable and should cause no problem. For example, between a moving train and a platform, the platform's frame of reference was usually preferred. What was, however, further noted was that students inadvertently placed themselves in this more natural frame and made value judgements about 'real' and 'apparent'-ness of motion. A related interesting response is quoted: 'Frame of reference is useful for understanding what [the] actual thing is. Feeling and actual thing are very different in science'.

The interview tried to probe students' intuitive justification for the real apparent distinction. One idea was that the platform's motion (relative to the child in the train) was not real because it was only the child and nobody else who saw it moving. The train's motion was real because after some time the train would not be there. (Note the unconscious adoption of the platform's frame.) When reminded of the relativity of motion, the interviewee retracted from her earlier stand only to come back to it in an unexpected guise. She remarked that, between the two frames, it was not possible to decide which motion was real and which apparent. For that purpose one must have a third 'neutral' observer who was aware of the motion of both the train and the platform. It seemed that, in the student's mind, relativity of motion of train and platform coexisted vaguely with the (implicit) idea of an observer at absolute rest (let us remind ourselves that this view is not as trivial and absurd as a physicist might be inclined to think now. It is, indeed, close to the original Newtonian conception). The real–apparent dichotomy was also seen to be justified on other grounds; for example, a train's motion is real because one can feel it, since its size is limited. The motion of a platform cannot be felt because 'the earth is so big that its velocity is negligible'!

Overall, students' free responses, particularly in the interviews, seemed to suggest that their belief in the correctness of AC(V) was more firm than the belief in their own justification for that conception. Consequently, instead of diagnosing their different fragile justifications, the alternative conception was tested *per se* in Q.6 and Q.8 of the forced-option test.

Alternative conception (VI) [AC(VI)]: Physical description through viewing

When students referred to motion relative to a frame, they sometimes seemed to mean the visual appearance of motion or the moving object seen (in a literal sense) from that frame ('frame' in the sense of AC(I) or AC(II) above). Thus a response (to Q.3(2) of the free-response test) like 'the child moving from door to window can be seen from the platform', suggests that the student ascertains motion by appearance rather than by transformation of velocities from one frame to another. Other indicators of AC(VI) were using 'vertical' in the sense of 'overhead' and equating line of sight with the trajectory of motion. The interview brought forth a most interesting response related to AC(VI). When questioned whether a distant tree (on the other side of the train) was in the platform's frame of reference, the student answered yes, but added the condition that the tree should be higher than the train so that its top could be seen from the platform!

The anthropomorphic connotation of the word 'observer' that seemed to underlie AC(VI) was not explicitly investigated in the forced-option test. Rather, we tested its indirect manifestation in students' equating physical description or measurement to an observer's viewing. Q.9 was designed to test whether the visual size of an object from a 'distant' frame was taken to be the measured size of the object in that frame. Q.10 and Q.11 tested the analogous notion for the speed of an object and for the trajectory of motion.

Alternative conception (VII) [AC(VII)]: 'Pseudorelativism'

In general, the results of the free-response test suggested that most students did not appreciate the use of the conceptual tool 'frame of reference' in physics, but they did have a rudimentary idea of relativity of motion, in that they realized that descriptions of motion vary among observers. When this notion is combined with AC(VI), an interesting kind of pseudorelativism emerges, namely that physical description of a phenomenon in a given frame of reference may not be unique but depends on how it is 'viewed'. Q.11 of the forced-option test investigates this 'pseudorelativism' in the context of the trajectory of a conical pendulum viewed by three different observers in a common (laboratory) frame of reference.

Test results

The forced-option test is given in Appendix B. The results are also summarized there, in terms of the percentages of students who agreed with, disagreed with, or gave no response to, the given statement. The correct responses are underlined. In this section, we interpret the results in terms of the alternative conceptions identified earlier.

Note that although for each question four choices were offered to students, the prevalence of alternative conceptions was found by collapsing the responses into two dichotomous categories: 'agree' and 'disagree'. The decision to do so was taken on the basis of pilot testing, which suggested that the two middle categories might be more a reflection of indecisiveness of personality rather than of lack of conviction about the alternative conceptions. Nonetheless, there were striking variations *between* questions, in the percentage of students responding in the two middle categories, which could only be interpreted as due to variations in the strength of conviction that the students had about these conceptions. A 'certainty index' was defined as the percentage of students responding with certainty, out of the total number who held that conception. As we discuss later, this index turned out to be strongly correlated with the prevalence of alternative conceptions.

AC(I): Frames behave like concrete objects that are physically fixed to bodies

Five different aspects of AC(I) were tested. These are listed in table 1, which shows that AC(I) is exhibited to a different extent in different contexts. For example, the idea of a frame smoothly 'changing course' along with the body it is initially moving with, is more natural than the idea of a tossing frame. The consistency between these two conceptions is seen in table 2 ($\chi^2 = 15{\cdot}8$, sig. $= 0{\cdot}000$).

Table 1. Aspects of AC(I): Treating frames as concrete objects.

Aspects of AC(I)		*Q.no.*	*Prevalence* (%)	*Certainty index* (%)
AC(Ia)	If the body to which a frame is initially 'attached' changes course, the frame also changes course.	Q.1d	71	71
		Q.2c	58	58
AC(Ib)	If two bodies collide, the frames moving with them merge.	Q.1c	49	39
AC(Ic)	If a body separates into two parts, the frame initially 'attached' to it also splits into two parts.	Q.3c	55	67
AC(Id)	If a body to which a frame is 'attached' disintegrates, the frame is destroyed.	Q.1e	51	56
		Q.3d	35	54
AC(Ie)	If a body changes its motion to match the motion of another frame that 'encloses' it, the frame initially moving with the body becomes identical with this other frame.	Q.2d	56	61

Table 2. AC(Ia): Idea of a frame changing course.

AC(Ia)		*(Q.2c) Frame tosses* Yes (%)	No (%)
(Q.1d) Frame (smoothly) changes course	Yes	50	22
	No	6	18

Similarly, the idea of a frame 'disintegrating' was more plausible in the airplane collision situation than in the situation where two trolleys merely separate. The consistency between the two ideas of frames 'splitting' and 'merging' was somewhat less ($\chi^2 = 6{\cdot}3$, sig. = 0·01), but still significant. Thirty-seven per cent of students held one idea, but not the other. Considering both Q.1c and Q.3c, 71% of students were seen to hold AC(I) in one or the other of the situations.

Yet another aspect of AC(I) was tested in Q.4b, in which students were asked about the 'temperature of a frame'. This statement was initially designed as a distractor, so it was surprising that 59% of students agreed with it. In addition, there was some consistency between responses to this 'temperature' question, and the ideas of a frame 'tossing around', and a frame 'splitting'.

To summarize, a large number of students think of frames as concrete objects. The tendency to do so may be more marked in some situations (e.g., the airplane problem) than in others (e.g., the trolley problem). Perhaps concretization is simply a convenient way of thinking, a prop that is often useful, that is readily abandoned when one's back is to the wall. But, such thinking is perhaps germane to other related alternative conceptions, like AC(II) and AC(III). Frames are merely a decoration, an inconsequential tiling of reality.

AC(II): Frames have local domains defined by the finite extension of the objects that they are 'fixed' to

Three different aspects of AC(II) were tested, as listed in table 3.

Question 4 had to do with frames attached to the sun, earth and the moon: three bodies which are known to differ greatly in size. This size difference was reiterated in Q.4c and Q.4d, and students were asked about the 'extensions' of frames attached to the sun, earth and moon. About 40% of students showed AC(II) in these situations, with high consistency between the two subquestions ($\chi^2 = 57{\cdot}0$, sig. = 0·0000).

Subquestions 5b and 5d suggested to the students that the co-ordinate axes of the frame had finite extension, limited by the extension of the ship in the question. Again, 35–41% of students agreed with these statements, with fairly high internal consistency ($\chi^2 = 7{\cdot}3$, sig. = 0·007). Further, there was a group of students (26%)

Table 3. Aspects of AC(II): Frames have local domains.

Aspects of AC(II)		*Q.no.*	*Prevalence* (%)	*Certainty index* (%)
AC(IIa)	Frames have finite extension	Q.4c	43	38
		Q.4d	40	47
AC(IIb)	Co-ordinate axes have finite extension	Q.5b	41	40
		Q.5d	35	41
AC(IIc)	Frames have a boundary	Q.5e	51	54
		Q.5f	33	30

who thought that the Z axis might go to infinity in the upward direction, but that in the downward direction it must be finite, presumably obstructed by the base of the ship, or by the seabed.

Question 5e dealt with the assumed 'boundaries' of a frame. Fifty-one per cent of students agreed that the airplane would go out of the boundary of the aircraft carrier's frame. The other question (Q.5f), which was designed to test for the boundary concept in an indirect way, showed no consistency with Q.5e, and was dropped from further analysis.

The three major aspects of AC(II), namely, extension of frame, extension of axes, and boundary of frame, had fairly high consistency between them (χ^2 of cross-tabulations ranged from 8·1 to 9·8). This consistency was of the same order as that seen within a single aspect of AC(II). Thus, it seems that in the range of situations covered by this test, AC(II) is a fairly coherent alternative conception.

AC(III): When small bodies are located on a larger body and moving relative to it, their motion is ignored, as they are 'part of the larger frame'

Two aspects of AC(III) were tested. These are shown in table 4. Additionally, cross-tabulation showed that 81% of students showed AC(III) in one or the other (slow or fast) situations, while 29% held it in only the more plausible slow-moving situation. Surprisingly, there was no correlation between aspects of AC(II) and AC(III).

Table 4. Aspects of AC(III): Small bodies moving on larger bodies are 'part of the larger frame'.

Aspects of AC(III)		*Q.no.*	*Prevalence* (%)	*Certainty index* (%)
AC(IIIa)	When the small body is moving slowly relative to the larger one	Q.7e	73	75
AC(IIIb)	When the small body is moving fast relative to the larger one	Q.7f	53	63

AC(IV): Phenomena belong to their particular frames

Two aspects of AC(IV) were tested. These are shown in table 5.

Question 6 involved two statements of the type 'the motion...*takes place* in frame...' (AC(IVa)). Sixty-eight per cent of students accepted one or the other of these statements. Our hypothesis was that those students who held this particular alternative conception would imagine that the motion of a stone dropped inside the train would 'take place in' the train's frame of reference, while the motion of a stone dropped outside would 'take place' in the platform's frame. The data bore out this expectation ($\chi^2 = 53·0$, sig. = 0·0000).

AC(IVb) was tested in Q.7b and Q.7c. These were again two direct statements of the type, 'The motion...is a phenomenon relative to (frame 1) but not to (frame 2)'. The hypothesis was that those students who held this alternative conception would believe that the given phenomenon could be observed from a unique frame: either the frame that the phenomenon 'took place' in, or 'the other' frame. Forty-

Table 5. Aspects of AC(IV): Phenomena belong to their particular frames.

Aspects of AC(IV)		*Q.no.*	*Prevalence* (%)	*Certainty index* (%)
AC(IVa)	'the motion...*takes place* in frame...'	Q.6f	59	57
		Q.6g	63	69
AC(IVb)	'The motion...*is a phenomenon relative to* (frame 1) but not to (frame 2)'	Q.7b	22	42
		Q.7c	34	60
		Q.7d	33	59

five per cent of the students were found to hold one or the other form of this conception.

Question 7d also probed AC(IVb), and was meant to provide an additional consistency check for Questions 7b and 7c. It helped identify a substantial core of students who thought that the motion of the ball *could not be* a phenomenon relative to both ship and shore. Thirty-three per cent of the sample showed AC(IV) in Question 7d. Nearly all of them (31% of the sample) also showed AC(IV) in Q.7b and Q.7c.

To summarize, the notion of phenomena being associated with particular frames seems to occur in a weak form in the majority of students. Thus, most students believe that phenomena 'take place' in particular frames, where the 'frame' in this description is the larger environment (like a train, or ship, or platform) in which that phenomenon occurs. However, 98% of the students responded correctly to a simple question on the relativity of motion (Q.7a). Most students also realized, at least in principle, that one phenomenon could be viewed from different frames (65%, from Q.7d). Nevertheless, there was a hard core of students (31%) who held this alternative notion in the strong form, who consistently expressed their belief that the given motion could not be considered as a phenomenon relative to two different frames.

AC(V): Some motions are real and some apparent

The results on AC(V) are summarized in table 6. The questions in this group referred to a supposed 'real' or 'actual' motion, as opposed to an 'apparent' motion. They were meant to test whether students inadvertently placed themselves in a natural frame of their choice, and consequently made value judgements about the 'real' and 'apparent'-ness of motion. A large number of students were found to do so.

Questions 6a and 6b, being related, must be interpreted together. Here, 72% of students answered in terms of real and apparent motion. Interestingly, the majority of such students (44% of the sample) placed themselves consistently in the platform's frame, although the trajectory of the stone was simpler in the train's frame. The subsequent questions 8a and 8f were framed in the anticipation that the platform's frame would be the more natural one for students. The percentage of students showing AC(V) in these situations was 76% and 72% respectively.

Looking at the consistency between the responses in table 6, we found a core group, 52% of the sample, who answered consistently in terms of real and apparentness of motion.

Table 6. Summary of results on AC(V).

AC(V): Some motions are real and some apparent		*Prevalence* (%)	*Certainty index* (%)
Q.6a and	'Actual trajectory' of stone is a	72	75
Q.6b	straight line or a parabola		59
Q.8a	Slanting motion of the rain is not 'real'	76	70
Q.8f	'Actually, the motion of the coconut is vertical'	72	82

AC(VI): Phenomena are ascertained through viewing

Three aspects of AC(VI) were tested, as listed in table 7. The questions in this group were statements ostensibly about frames of reference, but actually referring to viewing, of object size (Q.9), of speed (Q.10) and of shape of a trajectory (Q.11). Within a problem situation, the statements were logically interrelated, so the incidence of AC(VI) had to be inferred from consideration of pairs of questions. The salient results are summarized here.

Table 7. Aspects of AC(VI): Phenomena are ascertained through viewing.

Aspects of AC(VI)		*Q.no.*	*Prevalence* (%)	*Certainty index* (%)
AC(VIa)	Visual size of an object is the object's size in the observer's frame of reference	Q.9a	79	85
		Q.9b	62	59
		Q.9c	72	77
AC(VIb)	Visual speed of an object is the object's speed in the observer's frame of reference	Q.10a	79	67
		Q.10b	81	99
AC(VIc)	Trajectory of motion in a given frame varies depending on how it is viewed	Q.11a	51	63
		Q.11b	45	64
		Q.11c	80	72

Combined results from Qs.9a–9c (AC(VIa)) showed that a large percentage of students (70–80%) confuse measuring with viewing, as far as object size is concerned. However, the appeal of this idea depends to a considerable extent on the phrasing of the statement. Up to 26% of students gave logically inconsistent responses, probably because they changed their minds following a different phrasing of the statement (recall here that the students did the items in the given order, and were asked not to go back and change their earlier responses). For example, of the 79% students who showed AC(VI) in Q.9a, a good proportion

(26% of the sample) apparently changed their minds in response to Q.9b (this is also supported by the lower certainty index for Q.9b).

Questions 10a and 10b had to do with equating visual appearance of speed with measurement (AC(VIb)). A large percentage of students were found to do so, with high consistency ($\chi^2 = 25{\cdot}5$, sig. = 0·0000).

In AC(VIc) (shape of trajectory is determined by viewing), the results were similar, showing a high proportion of students who held the conception, but (as in AC(VIa)), a high proportion were inconsistent in their responses, implying that the appeal of the statements was dependent on phrasing. In particular, statement 11c presented the 'viewing for measuring' conception in a pseudorelativistic garb ('All three trajectories are equally correct...'). In this form, it was chosen by the majority of students (80%).

To summarize, a large number of students confuse measuring with viewing. The conception was found to be equally prevalent in questions related to object size, speed and shape of trajectory. Perhaps the constant use of the term 'observer' leads students to believe that it is physical viewing that is at issue in relativity of description of phenomena.

The large number of students showing AC(VI) in Q.11c led us to examine this particular statement, which refers to three observers in the same frame of reference. It seems that the students' choice of this statement might show the presence of yet another alternative conceptiòn, stated as AC(VII) below.

> *AC(VII): (Pseudorelativism) Descriptions in a given frame may vary among different observers, but they are equivalent, according to the principle of relativity*

Looking at the responses to Q.11b and Q.11c, we found that about 40% of the students, after answering Q.11b correctly, still agreed with the statement in Q.11c, which was framed in terms of the equivalence of three views of the trajectory of an object by three different observers in the same frame. Probably, these students held a pseudorelativistic view, in which the principle of relativity was equated to equivalence of different 'views' of observers located in a common frame of reference.

Relative prevalence of the alternative conceptions

Since we do not have a probabilistic sample, the results stated here in terms of percentages should be treated as specific to this group of students. With this caveat, some conclusions about relative prevalence follow naturally from the data.

A most surprising result in this study was the pattern of variation of the 'certainty index' between questions. In general, it turned out that the more prevalent alternative conceptions were also the more strongly held ones. The Pearson correlation coefficient between 'prevalence' and 'certainty' was a large as 0·77 (significance 0·000). This suggests that the origin of the more prevalent and strongly held conceptions might lie in fundamental and universal modes of cognition. On the other hand, such a strong correlation could arise from a diffusion mechanism, by which alternative conceptions were reinforced due to some kind of interactions among students. This question needs further research.

What we *can* state, based on this analysis, is that results on prevalence and certainty are mutually reinforcing. Both suggest that the alternative conceptions identified by us fall into two broad groups. AC(III), AC(V), AC(VI) and AC(VII) were very widely prevalent in our sample (held by 50–80% of the students), and they also appeared to be the more strongly held conceptions. On the other hand, AC(I), AC(II) and AC(IV) were less prevalent (held by 30–60% of students), and they were also less strongly held.

Conclusion

To sum up, a frame of reference in students' alternative view is a kind of chunk of space tied to a concrete body, bounded (but not closed) by the size and shape parameters of the body (with other smaller bodies located on it being part of the frame), sitting inside which an observer (human) 'looks' at phenomena. This picture allows rudimentary relativity in the sense that phenomena 'belonging' to a particular frame can nevertheless be 'looked' at from other frames 'outside', and these descriptions will differ from frame to frame. At one end, this picture co-exists with value judgements about 'real' and 'apparent'-ness of motion, which in the students' view does not negate relativity, but merely points to the existence of a 'neutral' frame. At the other end, the same picture (due to its anthropomorphic connotation) leads to an extreme relativistic view (pseudorelativistic indeed), namely the non-uniqueness of description of a phenomenon for different observers in a common frame of reference, and the supposed equivalence of these different descriptions by the principle of relativity. The overall impression that results from our analysis is that physics undergraduates tend to take 'frame of reference' as a decorative ploy with no explanatory purpose, and generally fail to show a metaconceptual understanding of 'frames of reference' as a tool for the proper formulation and exploitation of the physical principle of relativity.

Students' notions on relativity analysed in this paper relate to their alternative conceptions regarding frames of reference. Many alternative conceptions in Galilean relativity, however, arise from sources which do not depend on the artefact of 'frames of reference'. The investigation of these conceptions will be the subject of a future report.

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Appendix A. The free-response test

Q1. What are the things/ideas that come to your mind when you consider the term 'frame of reference'?

Q2. A ship is moving into a harbour. Let S be a frame fixed to the shore, and S' a frame moving with the ship. Describe any phenomenon relative to S and any other relative to S'. Can the first phenomenon be considered relative to S' and the second relative to S? Explain.

Q3. In the drawing below is shown a moving train and its surroundings. A vendor is walking on the platform. Inside the train, a child is running in the passage of a compartment. Let S be the frame of reference fixed to the platform and S' the frame moving with the train (Figure 1).

State if the following statements are true or false, and explain your answer briefly:

1. The vendor is walking in the S frame and the child is running in the S' frame.
2. The child appears at rest to the vendor.
3. The platform appears to the child to be moving.
4. The train's motion is real while the platform's motion is apparent.

Q4. Two open trolleys are moving with the same speed on parallel tracks, one behind the other. Let S1 be the frame fixed to trolley 1 and S2 the frame fixed to trolley 2. A boy in trolley 2 throws a ball vertically upwards. Will the motion be vertical relative to S1? Explain.

Q5. A child drags a long rod obliquely along the floor as shown in the figure. Consider two frames fixed to the rod: S1 has one axis along the rod. S2 has one axis along the direction of motion. An insect moves on the rod with uniform velocity relative to S1. Will the motion be uniform relative to S2? Explain.

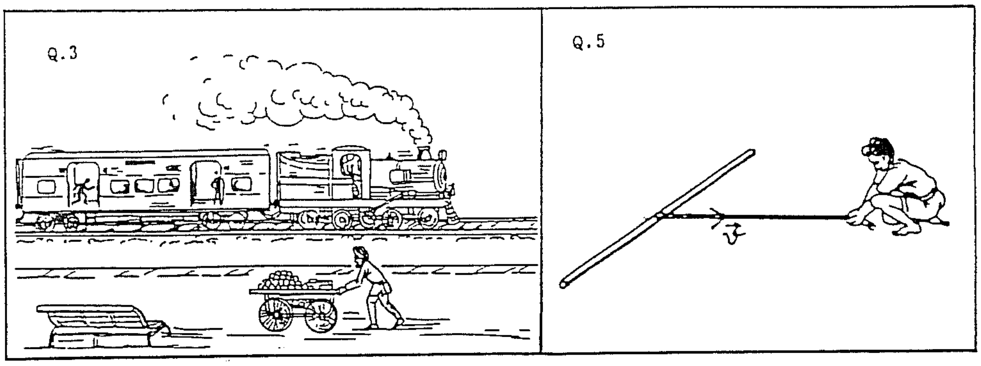

Figure 1. The free-response test.

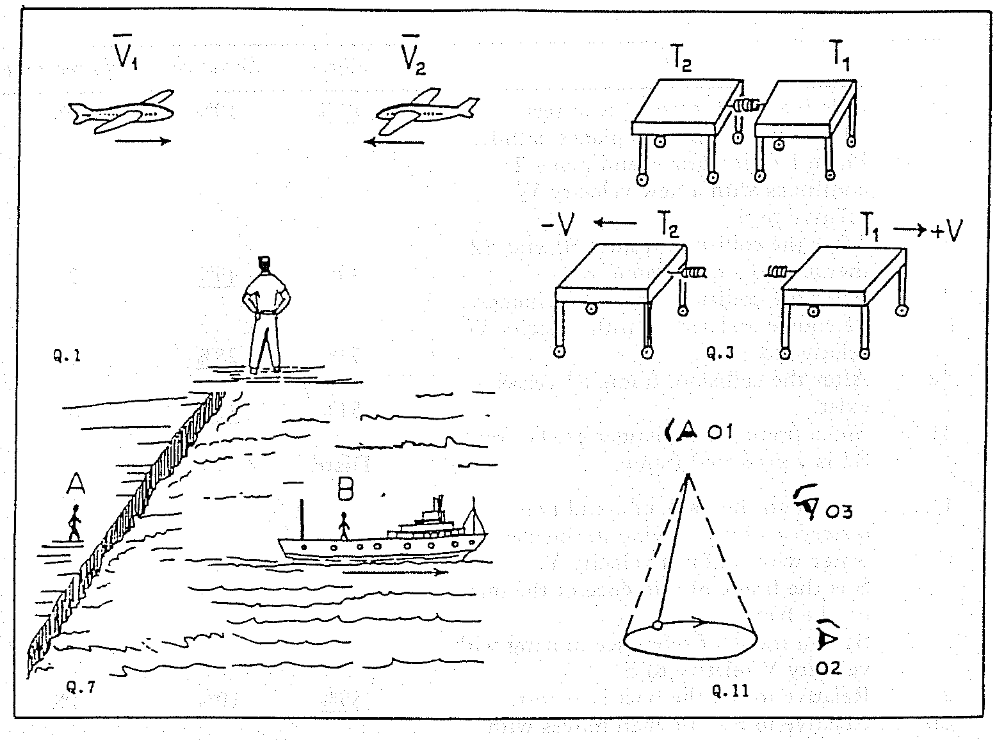

Figure 2. The forced-option test.

Appendix B. The forced-option test and results

In each of the 11 problem situations below, a series of statements are given. Some of these statements are followed by the options 'a, b, c, d'. Please select one of the four options (a, b, c, d) by circling it, using the following key:

(*a*) The statement is definitely true.
(*b*) Not sure, but the statement might possibly be true.
(*c*) Not sure, but the statement appears to be wrong.
(*d*) The statement is definitely untrue (or) it does not make sense.

Consider the statements only in the given sequence.
Do not go back to any question that you have already read.

(The results are summarized in terms of the percentages of students who agreed with, disagreed with, or gave no response to, the given statement. *The correct responses are underlined.*)

		Agree	*Disagree*	*No response*
Q.1	Two fighter planes are approaching each other with initial velocities V_1 and V_2 as seen by an observer on the ground (figure 2). S is the frame of the ground observer. S1 is a frame of reference moving with velocity V_1 relative to S. S2 is a frame of reference moving with velocity V_2 relative to S.			
1a	Relative to S1, plane 1 is at rest.	<u>80%</u>	19%	1%

		Agree	*Disagree*	*No response*
1b	Relative to S2, plane 2 is at rest.	<u>81%</u>	19%	0%
	After a while, the two planes collide. Plane 1 disintegrates and plane 2 continues with a new velocity V_2' relative to S.			
1c	After the collision, frames S1 and S2 merge into a new frame, S3.	49%	<u>49%</u>	2%
1d	After the collision, frame S2 changes its course and moves with velocity V_2' relative to S.	71%	<u>25%</u>	4%
1e	After the collision, frame S1 ceases to exist.	51%	<u>42%</u>	6%
1f	Since frame S1 no longer exists, frame S2 is a preferred frame.	Distr.		
Q.2	A man on the bank of a still pond observes a boat moving in the clear water with uniform velocity V. S is the frame of reference of the man on the bank. S1 is a frame of reference moving with velocity V relative to S.			
2a	Relative to S1, the boat is at rest.	<u>89%</u>	10%	1%
2b	Relative to S1, the man moves with velocity $-V$.	<u>86%</u>	13%	1%
	After a while, the man observes the boat entering the muddy water near the edge of the pond. The boat tosses around irregularly and eventually comes to a halt.			
2c	Simultaneously, frame S1 also tosses around and comes to a halt.	58%	<u>40%</u>	2%
2d	S1 and S now become identical frames.	56%	<u>42%</u>	2%
2f	Eventually, the boat moves with velocity $-V$ relative to S1.	<u>36%</u>	60%	4%
Q.3	Two trolleys (T1 and T2), held together by a spring, are at rest on the ground. S is the frame of reference of the assembly at rest. The spring is released and the trolleys move apart with velocities V and $-V$ relative to the ground (figure 2). S1 is a frame moving with velocity V relative to the ground. S2 is a frame moving with velocity $-V$ relative to the ground.			
3a	Relative to S1, trolley T1 is at rest.	<u>89%</u>	11%	0%
3b	Relative to S2, trolley T2 is at rest.	<u>89%</u>	10%	1%
3c	Frame S is split into two frames, S1 and S2.	55%	<u>44%</u>	1%
3d	After the trolleys are separated, frame S ceases to exist, and no observations relative to S are possible.	35%	<u>63%</u>	2%
Q.4	Let S_E be the frame of reference of the Earth, i.e., the frame relative to which			

		Agree	Disagree	No response
	the Earth is at rest. Similarly, let S_M be the frame of reference of the Moon and S_S be the frame of reference of the Sun.			
4a	The Sun's frame is at absolute rest.	Distr.		
4b	The temperature of S_S is higher than that of S_E or S_M.	58%	39%	3%
4c	Since the Earth is bigger than the Moon, the extension of S_E is greater than that of S_M.	43%	53%	4%
4d	Since the Sun is the biggest among the three, S_S has the largest size among the three frames.	40%	57%	3%
4e	The Sun's frame is the most convenient frame for the solar system.	Distr.		
Q.5	An aircraft carrier ship is moving uniformly on the sea. S is the frame of reference of an observer on the ship. The X and Y axes lie along the plane of the deck, and the Z axis is in the vertical direction.			
5a	All points on the deck have the same Z axis coordinate.	69%	25%	6%
5b	The X and Y axes of the frame S have finite extension, and they terminate at the edges of the ship.	41%	54%	5%
5c	The Z axis extends to infinity in the upward direction.	86%	11%	3%
5d	In the downward direction also, the Z axis extends to infinity.	61%	35%	4%
	An aircraft takes off from the ship and flies out into the distance.			
5e	The aircraft is initially in the frame of reference S, but soon gets out of its boundary.	51%	44%	5%
	After completion of its mission, the aircraft returns to the ship.			
5f	The trajectory of the aircraft in the frame S is a closed curve.	59%	33%	8%
Q.6	A train passes by a platform with uniform velocity. Let S_A be the frame of reference of observer A standing on the platform, and S_B the frame of reference of observer B sitting in the train. B puts his hand out of the window and drops a stone. B observes the stone to fall in a straight line, and A observes it falling along a parabola (ignoring wind effects).			
6a	The motion is in fact along the parabola as observed by A; what B sees is not the actual trajectory of the stone.	57%	41%	2%
6b	The motion is in fact along the straight line as observed by B; what A sees is			

		Agree	*Disagree*	*No response*
	not the actual trajectory of the stone.	29%	68%	4%
6c	A and B in fact do not observe the same phenomenon; they observe two different phenomena.	52%	47%	1%
	Simultaneously with his other hand, B drops a stone inside the train.			
6d	The trajectory of this other stone as observed by A (assume the windows to be large and transparent), is parabolic.	59%	40%	2%
6e	The trajectory of this other stone as observed by A, is a straight line.	41%	55%	4%
6f	The motion of the first stone takes place in frame S_A.	59%	38%	4%
6g	The motion of the second stone takes place in frame S_B.	63%	33%	4%
6h	The actual motion of the second stone is as observed by B, and not as observed by A.	39%	56%	5%
Q.7	A ship is moving uniformly away from a shore. Let S be the frame of reference of an observer A on the shore, and S′ the frame of reference of an observer B standing on the ship. A pole is fixed on the deck of the ship.			
7a	To A, the pole appears to be moving away, while to B, it appears stationary.	98%	2%	0%
	B rolls a ball along the deck.			
7b	The motion of the ball is a phenomenon relative to S but not to S′.	22%	77%	1%
7c	The motion of the ball is a phenomenon relative to S′ but not to S.	34%	65%	1%
7d	The motion of the ball is a phenomenon relative to both S and S′.	65%	33%	2%
	The observer B now walks along the deck, towards the pole.			
7e	For the observer B, the ship is stationary.	73%	22%	5%
	Next, another observer standing on the ship fires a bullet.			
7f	Relative to the bullet's frame of reference, the ship is stationary.	53%	44%	3%
Q.8	A train is moving uniformly. Outside, it is raining. For an observer A on the ground the rain appears to fall vertically. For an observer B inside the train, the rain appears to fall in a slanting direction.			
8a	The rain actually falls vertically; its slanting is not real.	76%	23%	1%
	Relative to the train, the entire air mass outside moves in the opposite direction. Assume this motion, or 'wind', to be uniform; ignore non-uniformities close to the train.			
8b	The wind described above causes the slanting motion of the rain.	61%	35%	4%

		Agree	Disagree	No response
8c	Therefore the slanting motion of the rain is real.	32%	63%	5%
	A coconut drops from a tree outside the train. The wind effects on the coconut are naturally negligible.			
8d	The train observer also sees the coconut dropping vertically.	40%	58%	3%
8e	For the train observer the coconut does not drop vertically downwards, because of relative motion between the tree and the train.	70%	25%	5%
8f	The observer in the train sees the coconut falling a slanting (parabolic) path, but this motion is not the actual motion. Actually, the motion of the coconut is vertical.	72%	23%	5%
Q.9	One bus is standing at the top of a steep mountain road, while another similar bus is standing at the bottom end of the road. Observer A is located next to the top bus, and observer B is located next to the bus at the bottom. To A, the bus at the bottom looks smaller than the bus at the top. To B, the bus at the top looks smaller than the bus at the bottom.			
9a	The length of the bottom bus, as measured in A's frame of reference, is smaller than the length of the top bus.	79%	18%	3%
9b	The length of the top bus, as measured in B's frame of reference, is the same as the length of the bottom bus.	35%	62%	3%
9c	From either observer's frame of reference, the measured length of the bus at the other end, is contracted from its true length.	72%	22%	6%
Q.10	An airplane is flying high in the sky, and its meters show it to be moving at 2000 km per hour. To an observer on the ground, however, it appears to fly rather slowly. In comparison, a bird in the sky seems to move much faster.			
10a	In the frame of reference of the ground observer, the speed of the airplane is much less than that of the bird.	79%	18%	3%
10b	The actual speed of the airplane is much greater than that measured by the ground observer.	81%	14%	5%
Q.11	An experiment with a conical pendulum is set up in the laboratory (figure 2). The motion of the bob is observed by three observers, O1, O2 and O3. O1, viewing the motion from a point directly above the point of suspension, observes			

		Agree	*Disagree*	*No response*
	the bob to move along a circle. O2, viewing it in the plane of motion of the bob, finds that it moves along a straight line. O3, viewing the motion obliquely, finds the trajectory to be somewhat oval-shaped. (Figure B.5 shows such an oval-shaped trajectory.)			
11a	The circle and the straight line are equally correct descriptions of the bob's motion; the third (oval) trajectory is a mere illusion.	51%	46%	3%
11b	The circular trajectory observer by O1 is the only correct description of motion of the bob in the lab frame. The other descriptions are modified by effects of viewing.	53%	45%	2%
11c	All three trajectories are equally correct descriptions of the bob's motion in the lab frame, since the trajectory depends on the observer looking at the motion.	80%	17%	3%