EXTENDING THE MEANING OF FRACTION NOTATION: A TEACHING EXPERIMENT

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The National Council for Education Research and Training has recently concluded a major review of the school curriculum. One of the issues debated is whether the topic of fractions should be taught at the primary level. The Position Paper of the National Focus Group on the Teaching of Mathematics calls for a careful reconsideration of the content of the topic of fractions, citing the fact that the use of fractions in real world computations have largely disappeared with the adoption of metric units and the use of decimal numbers. Given the difficulty that a number of students face in mastering this concept, the Position Paper recommends that less emphasis be placed on operations with fractions at the primary level (NCERT, 2006). Others have taken a more radical view, and have advocated removal of the topic from the primary school curriculum (Verma and Mukherjee, 1999). In this paper, we shall attempt to indicate the importance that the concept of fractions has for subsequent learning, especially for the learning of the core concepts of, what has been described as ‘multiplicative thinking’.

A large number of research studies have focused on the role of multiplicative thinking in elementary mathematics and the difficulties that it poses for children. Research indicates that many children at the end of primary school frequently employ additive rather than multiplicative thinking and are hence underprepared for middle school arithmetic in which percentages, ratio and proportion play a central role. Children who are the additive reasoners interpret changes to the values of the quantities as additive transformations, and therefore, employ additive compensations even in situations in which multiplicative compensations are required (Harel and Behr, 1993). Multiplicative understanding demands that children learn two different aspects: Firstly, children need to know which situations can be handled by multiplication or division. Secondly, they need strategies to solve these multiplicative problems.

Many have identified multiplicative thinking as the core part of middle school arithmetic. Vergnaud has developed a framework for understanding the conceptual development of arithmetic in middle school by relating all the concepts required for developing number sense and multiplicative reasoning under the term Multiplicative Conceptual field (MCF). The concepts that constitute MCF include multiplication, division, fraction, ratio and proportion and relationships among them. It is commonly acknowledged that there is much interconnection between these concepts. However, translating the understanding of the complex conceptual web into an effective teaching learning sequence remains an unsolved problem. In this study, we report preliminary results from a teaching experiment that attempts to prepare students for multiplicative thinking by extending the meaning and use of the fraction notation.

The study reported here is a part of the Curriculum Development project conducted at Homi Bhabha Centre for Science Education (HBCSE), Mumbai. A mathematics learning camp was organized in the summer vacation for about 40 students studying in the Marathi medium, who had just completed grade four and were about to enter grade five. The camp consisted of 19 days of instruction with 90 minute lessons each day. All the lessons were video recorded. Data was also collected in the form of worksheets and pre and post tests. In this poster, we shall examine qualitative data from the classroom video recordings and worksheets to assess the effectiveness of the teaching learning devices used such as, specific problems and tasks, diagrams used to represent key concepts and classroom discussion.

FRAMEWORK OF THE STUDY

In the primary grades, students in Indian schools are typically introduced to fractions in the context of part-whole relationships often represented by the 'area model', that is as shaded portions of shapes. While this gives students an understanding of simple fractions, students typically do not develop an understanding of more difficult concepts such as equivalent fractions, improper fractions and operations on fractions.

In this study we aimed at extending the meaning and use of the fraction notation in a different direction. The first extension was the interpretation of a fraction as an operator, i.e. understanding the effect of multiplying a number by a fraction. The second extension was the use of the fraction symbol to denote the result of a division operation. Although these extensions are typically introduced in the middle school, they are used largely implicitly. Since these are key concepts underlying multiplicative thinking, we advocate a more careful treatment of these concepts. The focus of our study was to develop a teaching learning sequence for introducing these concepts explicitly through extensions of the meaning of the fraction symbol.

Some other beliefs, derived from our ongoing experience of curriculum development, motivated the approach that we adopted in the study. By the end of primary school, students typically have developed a robust understanding of small whole numbers. For many of them, numbers begin to take on a concrete existence. Many, though not all, implicitly recognize simple multiplicative relations between numbers. The approach we adopted began by bringing these relations into explicit focus, while simultaneously developing tools in the form of key diagrams and symbolic notation to characterize these relationships. In this report, we shall examine the relationship between students' prior knowledge, the devices introduced in the form of diagrams, symbolic notation, vocabulary and concepts, and the change in the ways in which students interpreted and used the fraction notation as the instruction progressed.

USE OF DIAGRAMS

In the initial classes, students were introduced to multiplicative relations between whole numbers. At first, this was framed through a concrete situation: how many bananas (Rs 2 each) can you buy for the price of one apple (Rs 12)? This was supported by using a key diagram such as shown in Fig. 1a. The attention of the students was called to the relation between the two numbers. Answering the question of 'how many times' a bigger number was of a smaller number was simple, while the answer for the question in reverse was not known to the students. However students were sure that a definite relation existed in the reverse direction. In order to develop to an understanding of this relation, the diagram was explicated as in Fig. 1b. Now students were able to use their prior knowledge of fractions as part-whole relations to arrive at the conclusion that 3 is ¼ times 12. Students quickly replaced, over the course of a few examples, the detailed diagram by the simpler one as shown in Fig. 1c, and easily found and expressed multiplicative relations for different pairs of numbers regardless of whether the bigger number was shown on the right or on the left. (At this stage the numbers used were such that the bigger number was a multiple of the smaller number.) This representation remained popular with the students till the end of the camp.

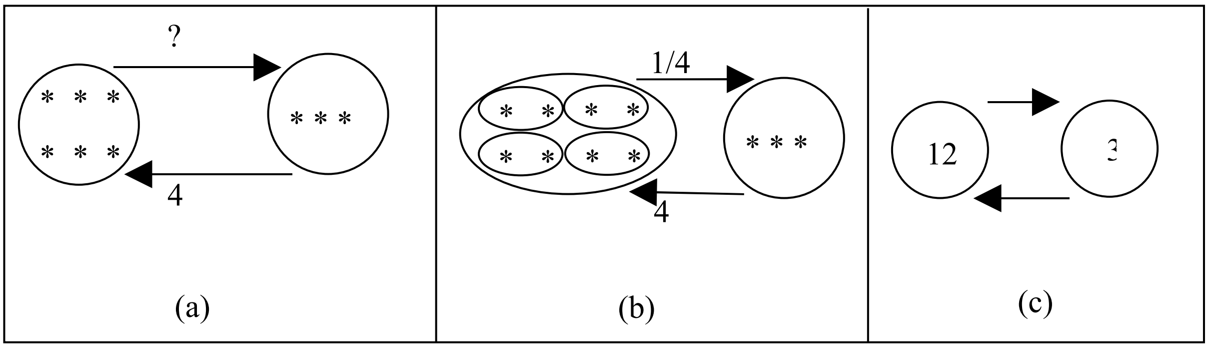

Figure 1

For more difficult tasks, such as finding 1/3rd of 39, or finding how many times 18 is of 45, a different representation was suggested by the teacher (See Fig. 2).

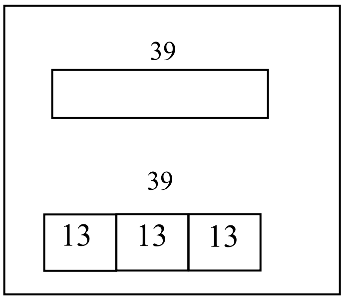

Figure 2

This representation, accompanied often by extensive verbal discussion was used by students to solve such problems as finding 2/3rd, 4/5th, 3/4th and so on for given numbers. The students were already familiar with dividing a shape into equal parts in the context of learning fractions. Hence they had access to this representation as well as the accompanying vocabulary, as was indicated by the verbal justification that they provided. For example, '*to find 3/4th of 60, I divided 60 into 4 equal parts and to find ¾ I took 3 parts of ¼ which comes to 45*'.

VERBAL REASONING

Although the students easily understood the relation of times and were able to verbalize it as well as represent it through a diagram, associating this idea with the multiplication operation needed explicit discussion. One of the tasks posed to the students was to fill in the blank in the problem 12 _______ 24, with an operation sign, a suitable number and an '=' sign. Students were able to complete this as 12 × 2 = 24. They were then posed the inverse problem: 25 _____ 5. The initial solutions proposed were '÷ 5' or '× 5' when they were urged to use the '×' sign. Eventually one of the students offered this argument, '*if we make 5 equal parts of 25 and take one part of it we get 5, hence5 is 1/5 times 25 (5 hi 25 chya 1/5 pat aahe), so 25 × 1/5 =5*'. This was a new idea to the other students, who accepted it after some discussion. Acceptance of this idea was facilitated, among other things, by the word used for describing the multiplicative relation, namely '*pat*', meaning '*times*'.

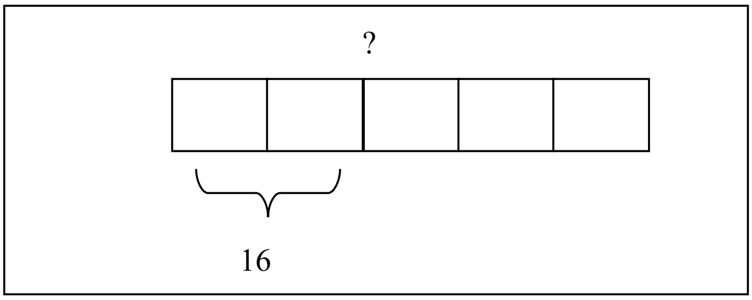

Fig. 3

A further extension of the problem described above was to fill in the blank in the question ____ × 2/5 = 16 by a suitable number. This was solved by the students by asking what is the whole, which if we divide into five equal parts and take two parts we get 16. When the teacher probed further, the students also offered the diagram shown in Fig. 3 to explain the solution to the problem. Students explained saying that as each part is equal, each part is 8 and hence the whole is 40. Here we see a combination of verbal reasoning and the use of a key diagram.

Although the students often presented relevant and reasonable examples of verbal reasoning, some forms of verbalization were persistently problematic for them. In the initial lessons on multiplicative relations between numbers, one of the tasks was to express these relations verbally. For example the relationship between 15 and 90 was expressed in the following two ways.

15 hi 90 chya 1/6 pat aahe. (15 is 1/6th 'times' 90)

90 hi 15 chya 6 pat aahe. (90 is 6 times 15)

Several students wrongly inverted these relationships, writing '15 is 6 times 90'. Moreover, these errors were persistent. The students of course, understood the relation perfectly as indicated by their responses whenever the problem was presented in diagrammatic form. In one classroom episode, when the teacher complained about this error to the students, the students laughed at their own mistakes. One explanation for this error could be that the cognitive processing of these sentences makes excessive demands in comparison to the processing of diagrams. It may be noted that the diagrams contain an arrow pointing to the target number, i.e., the result of the multiplication, while the direction of the verbal sentence is opposite. Hence the students' error may indicate a dominance of the diagrammatic over the verbal representation.

FRACTION AS NOTATION FOR DIVISION

One of the tasks presented to the students was to write as many multiplication facts as they can which correspond to (i.e. express the same information as) a given division fact. For the division fact $12 \div 6 = 2$, the students not only wrote the multiplication facts $6 \times 2 = 12$ and $2 \times 6 = 12$, but also the multiplication facts with fractions: $2 \times ½ = 6$ and $12 \times 1/6 = 2$. They also noted that if we want to retain the same answer as the original division fact, then only one multiplication fact fulfils this condition, namely, $12 \times 1/6 = 2$.

This response by the students prepared the ground for the teacher to introduce the fraction notation for division. Students recognized that $36 \div 6$ and $36 \times 1/6$ are different versions of the same problem and have the same answer '6'. When the teacher asked why this was so, the students offered different explanations. Most of the explanations were adequate and made reference to the action of making equal parts, which is common to the part whole model of fractions and the partitive model of division.

The teacher next introduced the idea of using the fraction notation to denote the result of a division operation. However, the students accepted this idea only for the case of incomplete division, for example, $1 \div 2 = ½$ and $2 \div 4 = 2/4$. They were unwilling to accept this notation for division facts such as $4 \div 2 = 4/2$, where the result is a whole number. When this was probed further by the teacher, it was found that the students were not willing to accept 4/2 as a fraction. The part whole model that they had, together with the area representation created difficulties for understanding improper fractions. This proved to be such a strong block that at this point of the instruction, the teacher had to revert to the topic of fractions and develop the concept of improper fractions.

CONCLUSION

The fraction concept and notation can function as a key tool in strengthening and developing children's intuitive understanding of multiplicative relations between numbers. Children already have an understanding of fractions as part-whole relations and can extend this meaning to express multiplicative relations using fractions as operators. As indicated by the students' responses and classroom discussion, this extension is not trivial and may represent genuine learning. The use of the fraction notation for the division operation plays an important role in the middle school curriculum and beyond. Hence, students' ability to understand this use of the fraction notation can be unduly curtailed if the fraction concept is poorly developed in the primary years, especially with regard to the concept of improper fractions. Careful instruction aimed at extending the meaning of the fraction notation to denote the division operation and to the concept of ratio, may help in strengthening students' multiplicative reasoning. How this may help in subsequent learning of middle school mathematics, in problem solving and in the learning of algebra needs to be further investigated.

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