VIDEOCASES AS A FORM OF DIDACTICAL INTERVENTION IN A COLLEGE PROOF CLASS

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INTRODUCTION

It has long been documented that students have difficulty producing proofs (e.g Alibert and Thomas, 1991, Knipping 2004). Among the reasons for this difficulty, one that seems particularly significant is a difficulty coordinating informal and formal aspects of the proof. (e.g. Almeida, D. 1996, Hoyles and Healy 1999, Raman 2003) For instance even students who are able to give good arguments for why a claim is true may have difficulty writing a formal proof (Schoenfeld, 1991).

The difficulty of producing proofs is so difficult that at the college level, many American universities have now created special “Introduction to Proof” courses aimed primarily at helping students write mathematical proofs before they take proof heavy courses like analysis and abstract algebra. However, there seems to be no consensus on what this course should be like or how to best help students. For instance at Rutgers University, where there might be six sections of this course taught a semester, there will be six completely different approaches. And the results of these courses, measured by student feeback and results in later courses, seems to vary considerably.

This study came out of a desire to help the faculty who teach these “Introduction to Proof” courses and the students who struggle through them. At the center of the study is a set of videocases of “experts” producing proofs of theorems that students tend to find particularly difficult in this type of course.

The use of videos in mathematics education is not new. Videocases have been used successfully in a variety of settings, such as preservice teacher training (e.g. Beck, King, and Marshall, 2002; Ozsuz, Savenye, & Middleton, 2003). To our knowledge, this approach has not been used for helping mathematics students learn to prove, or for that matter, to highlight any sorts of aspects of mathematical thinking.

The videos described in this study involve graduate students or advanced undergraduate students working on fairly difficult, but typical, proofs from this course. As these “experts” work on the proofs, they talk aloud, so students can see where they get stuck and how they get past their difficulties. In particular the “experts” are instructed to talk about their informal, intuitive ideas behind the proof and describe, as much as they are able, how they can express those ideas formally.

As students watch these videos, having already attempted the proofs, they see examples of the “behind the scenes” thinking that is typically hidden from textbooks and classroom discourse. The hope is that by making this important aspect of mathematical thinking more visible, students will be able to identify where they get stuck and how to get unstuck, and this in turn will help them produce proofs on their own.

METHODS

The setting for this study involved students from Introduction to Proof Courses at two major universities in the United States. As noted above, this type of course, which seeks to bridge the more computational lower division courses with the more theoretical upper division courses, are considered difficult by many students. In the last few years, increasing attention has been paid to how to improve this type of course (E.g. Alcock, in press). This study contributes to that literature.

The original list of questions for the study are given below:

1. Prove: For all intervals A, B, C , if $A \cap B \cap C \neq \emptyset$ then one of the intervals is contained in the union of the other two.
2. Let f : $B \to C$, g: $A \to B$ and h : $A \to B$ be functions satisfying $f \circ g = f \circ h$. Prove or disprove:

(a) If f is surjective then $g = h$.

(b) If f is injective then $g = h$.

3. An integer n is said to be a square if there is an integer k such that $n = k^2$ and is said to be square-free if it has no divisor bigger than 1 that is a square. Use the well-ordering principle to prove: Every positive integer can be written as the product of a square and a square-free integer.
4. Let A, B, and C be sets. Prove or disprove:

(a) If A – C = B – C then A = B.

(b) If A – C = B – C and C – A = C – B then A = B.

These questions were piloted with students at Rutgers University and Georgetown University in Spring 2006, and constitute the bulk of the data in this report. The list of questions has recently been expanded to ten, and will be piloted in Fall 2006 at Georgetown University.

The videos can be used in a number of ways, and these will all be discussed in the full version of the paper. In all cases, students watch a video of either a fellow student (who has successfully completed the course recently) or a graduate student who has been enlisted to work on these proofs. The students in the study are asked to do the proofs on their own and then watch view videos and reflect on their experience with the proofs. They watch the videos as a class and the teacher stops the video whenever she or the students would like to comment. In some cases, if the students can see where the proof is heading, the teacher stops the video and allows the students to continue with the proof on their own.

DISCUSSION

While the idea of using videos in instruction is not new, the idea of using videos with students in a math course (in particular a university level math course!) has not been exploited. We are in the beginning stages of understanding how video can best be used as an instructional aid. The talk will include data from two pilot studies where students watched and discussed these videos in a classroom setting.

The main finding from the studies is that video appears to be a very powerful way of teaching mathematical thinking skills that typically go unaddressed in mathematical classrooms. Some of the questions that came up from students watching the videos included:

1. When do you use cases? How many cases do you use?
2. How did this person know that the proof would be direct instead of indirect?
3. How do I convert the pictures in my head into a proof that would be acceptable in class?

These are all wonderful questions and it seems important to put them front and center in the classroom. In a typical class where a teacher presents material and students go home to work on exercises, these questions would not have a place inside the classroom. They might occur to students while working on their own, but there would not necessarily be instructional time to address them.

Using videos to make mathematical thinking the central point of the classroom shifts the role of class time from presenting mathematics to making mathematical thinking explicit. This is a dramatic shift, and one which we now have reason to believe could be beneficial to students trying to make the large jump from fairly rote computations to real mathematical thinking. However we have only begun to scratch the surface of this new technology and we look forward to discussing how this medium could be used in other settings and at other grade levels.

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