DEVELOPMENT OF LOGICAL THINKING AMONG CALCULUS STUDENTS THROUGH A DISCOVERY APPROACH

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The development of logical thinking among calculus students is one of the forgotten commitments of the origin of the first generation calculus reform movement of the eighties. The memorable text "Toward a Lean and Lively Calculus" contains a report by Susanna Epp (Epp, 1986) entitled, The Logic of Teaching Calculus, which to some degree provides a blueprint for the transformation of the language of the subject so as to assure a systematic and sustained development of logical thinking during the course. Epp's presentation in (DIMACS, 1996) suggests that the efforts at teaching logic have been largely transferred out of calculus into discrete mathematics courses. However, discrete mathematics courses at many colleges are not prerequisite for calculus courses and directed largely to the liberal art student population, engineers and scientists taking calculus are separated from the benefits of logical thinking, and as our project shows, leads to dramatic lowering of the capability of understanding fundamental concepts of calculus such as limits of sequences and functions. Consequently it is the responsibility of Teacher-Researchers in calculus to create JiTI (Just in Time Instruction) of logical components needed for the successful dealing with calculus concepts such as negation of simple quantified statements, clarity in understanding the difference between the conditional if…then… and the equivalence " if and only if".

PROBLEM BEING ADDRESSED

The purpose of the current presentation is to describe a Teaching-Research method for developing logical thinking skills among calculus students simultaneously with the investigation of its effectiveness as part of the project Indivisibles in Calculus Instruction, (NSF/ROLE #0126141). The interests of Teacher-Researchers in understanding the logical thinking of calculus students arose as the result of two difficulties encountered by students in negating the definition of the limit of a sequence or of a function– one of the components needed for robust understanding of the concept; and in adapting to a Moore discovery method (Mahavier, 1999), where logical thinking is the foundation of the process of inquiry, hence, its pedagogy requires from students the ability to negate the definitions of a limit, proper utilization of if…then… structure, as well as ability of basic mathematical reasoning techniques.

THEORETICAL FRAMEWORK.

The study was conducted through the Teaching-Research cycle methodology of design, implementation, assessment and analysis of the data, re-design. TR methodology enables the teacher to investigate thinking of students simultaneously with teaching. The results of the investigation can be immediately introduced into teaching to improve the process of learning.

In particular, we have investigated the utility of Moore Discovery method for a Bronx community college student population. The Moore Discovery method utilizes the instructional sequence whose emphasis on (1) precision of language and (2) intended moments of understanding and logical deductions require that the teacher-researcher carefully guide and monitor the learning of the students and systematically act on the developing learning trajectory or note the difficulties and find means to address them. The teacher-researcher has to determine the cognitive distance between successive problems of the instructional sequence so that they are challenging but not impossible. The guidance of the student through the carefully designed cognitive challenges helps the teacher-researcher to facilitate continuous improvement of student's understanding and mastery of the concept in question.

The Research Questions posed for the present study are:

What is the nature of logical thinking acquired through the employment of the experientially based logical instruction using the Just-in-Time approach ?

How can the instructional sequence designed on the basis of the Moore discovery method be effectively utilized in community college classrooms? What are the difficulties that are encountered in the process and how can these be addressed through the Teaching-Research methodology?

RESEARCH DESIGN AND PROCEDURES

The Teaching-Research methodology is particularly useful for investigations of the nature and effectiveness of the innovative instructional sequences, which as (Wittman, 1999) asserts are in dire need of methodological development. Two versions of the geometrical definition of the convergence of a sequence are used as the bridge to carry students from an intuitive visual representation to the logical formalism required of the precise Weierstrassian definition of convergence. The instructional sequences incorporating JiTI of basic logical structures will be presented, and its\effectiveness discussed in terms of student capability to negate the geometric definition of the limit.

In particular we will present the analysis of student errors on the negation of single quantifier statements, which together with a similar analysis of student errors in checking the validity of syllogisms indicate that the proper conceptual instrument to frame the question of the development of basic logical skills is that of the schema of thinking. Schema of a concept is a network of the relationships relevant in reaching the understanding and mastery of the concept. The stages in the development of a schema were analyzed by many authors (e.g. Piaget and Garcia, 1989; Shuell, 1993). We will demonstrate the that students' errors in the construction of negation and understanding of syllogisms can be understood as different stages in the development of the schema for the relevant concept

The analytic definition with its required quantifiers are known to be difficult for students, and these difficulties are successfully bypassed by the precision of the language scaffolding the development through two geometric definitions. Scaffolding is present throughout the course - building on student intuition is a necessary condition for instructional material as well as classroom discourse. Hence, since all students have had some exposure to the real numbers in prior mathematics classes, and can join in classroom discussions about the set of real numbers, it is used as the basis for the course and is systematically developed, so that students' knowledge of the real numbers prior to the class and at its completion are markedly different. The development of real numbers in the entire first row below (Figure 1) is free of any use of acronyms (such as lub or glb – which are used here for the mathematically sophisticated reader and the restrictions of space) and logical symbols. It is the use of precise language in the instructional materials, and in the classroom discourse that develops the logic and the required precision in thinking.

Real numbers → sets of real numbers → lub/glb

↕ ↕ ↕

Sequence → range of sequence → limit

Figure 1

Note that in the first row, students disclose several misconceptions and this provides the opportunity for regular assessment of these misconceptions throughout the semester. Instruction of the second row is carefully coordinated with the developed concepts of the first row and the essays and assessments serve to reinforce the close relationship and develop the required connections. The first row also provides the opportunity to introduce mathematical definitions and an axiom. (Axiom 1 states: If M is a set of numbers that is bounded above, then either there is a largest number in M, or there is a smallest number larger than every number in M. If M bounded below, then there is a smallest number in M, or there is a largest number smaller than each number in M). The level of difficulty throughout the course is regulated, so that the problems are challenging but not impossible, and whenever a new concept is introduced, the problems are such that students can always enter the thinking required in the domain of the problem (examples given at the presentation). Once students enter into the domain, the level of difficulty is gradually increased to more complex problems, which lead in the development of the schematic described in Figure 1 above.

The development of logic in the concept of the limit of a sequence proceeds as shown in Figure 2 below. The knowledge of finite/infinite, bounded/unbounded sets of real numbers and the determination of the "lub/glb" is used as the base for this development. A sequence, is introduced as a function with the set of natural numbers as its domain and forms a connection between the range of a sequence and its underlying set of real numbers. The projection of the sequence on the vertical axis gives a visual representation of this set and the concept of the "neighborhood of the limit of the sequence" begins to take form through an epsilon-band or simply as a neighborhood of the limit. The shift of attention from the dynamic approach of the sequence to its limit, now in the context of the range of a sequence is followed by an enquiry of the nature of these epsilon-bands or neighborhoods – what is the restriction on them, how does a change in neighborhood affect the vertical line in the second geometric definition, and what is the relationship between the width of the band and the location of the vertical line. Discussion of the logical components in class and in mathematical essays at home occurs on a purely geometric basis through language and the reasoning is understood and developed without the symbolic use of any quantifiers. The geometric definitions develop the coordination between the visual/geometric and analytic/mathematical components and it is then the role of the analytic definition to provide the means to actually determine the location of the vertical line in terms of the distance between the two horizontal lines.

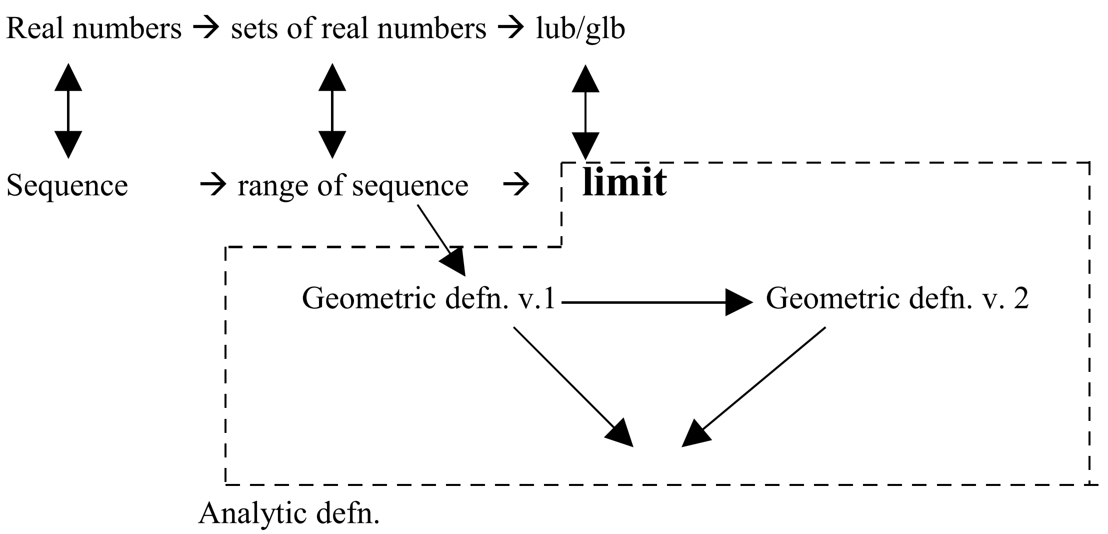

Figure 2

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