UNDERSTANDING ABSTRACT DEFINITIONS: GRAPHS, A CASE STUDY

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INTRODUCTION

The purpose of this paper is to report on our research into the factors which strongly influence the formation of different concept images when a student encounters a new mathematical definition. We investigate the initial obstacles to understanding a new mathematical definition as and when it is encountered. The questions which initiated our inquiry were the following:

1. What are the factors which influence the formation of pictorial, symbolic mental images triggered by a new mathematical definition? How do these factors influence their ability to apply the definition when they are confronted with a problem situation immediately after they had encountered the definition?

2. What are the initial obstacles which arise in conceptualising the definition immediately after a student encounters a new, abstract, mathematical definition?

3. Are the students able make use of visual and verbal cues implicit in the words used in the definition?

RESEARCH FRAMEWORK

Concept image and concept definition: In their foundational work, Vinner and Tall (1981) have provided a framework for understanding how one understands and uses a mathematical definition. According to Vinner and Tall, to each mathematical concept, a concept definition and a concept image are associated.

Concept image is the total cognitive structure associated with the mathematical concept in the individual's mind. Depending on the context different parts of the concept image may get activated; the part that is activated is referred to as the evoked concept image.

The form of words that is used to describe the concept image is called the concept definition. This could be formal and given to the individual as a part of a formal theory or it may be a personal definition invented by an individual describing his concept image. A potential conflict factor is any part of the concept image which conflicts with another part or any implication of the concept definition. Factors in different formal theories can give rise to such a conflict. A cognitive conflict is created when two mutually conflicting factors are evoked simultaneously in the mind of an individual. The potential conflict may not become a cognitive conflict if the implications of the concept definition does not

become a part of the individual's concept image. The lack of coordination between the concept image developed by an individual and the implication of the concept definition can lead to obstacles in learning.

RESEARCH METHOD

Our experiments consisted of tests, interviews and group discussions on two samples of students. This allowed us to arrive at certain conclusions regarding the nature of the concept images formed by the students. Our test items were based on basic notions of graph theory.

DISCUSSION AND CONCLUSION

Based on our experiments, we observe the following: Many students (70%) of the first sample have drawn the coordinate axes in response to questions based on the definition of graph. The word graph seems to have evoked the concept image of the Cartesian coordinate system and that of the graph of a function. Their prior knowledge of polygons with vertices and edges has probably led them to believe that the edges of a graph have to be straight and cannot be curved.

Their understanding that Cartesian product of two sets consists of ordered pairs has caused conflict with the definition of edges. Many students seem to have a concept image of an oriented edge as understood from their written responses and interviews.

It appears that the notion of a directed edge is more naturally conceived than that of an unoriented edge.

Our experiments strongly suggest that the labels used to describe the concept being defined in a definition plays a pivotal role in understanding of a mathematical definition. The figural representation of the concept is dependent upon these labels rather than its mathematical content (such as relations among the mathematical components) that the definition engenders. If the label used in the new definition is the same that which has been used in a different context, the figural representation of the previously learned concept seems to influence the formation of mental images associated to the new concept. The resulting figural representation was complicated, involving a blend of features of the old and the new. Such complicated mental images fail to reflect the salient features of the new definition to be of much help. This is perhaps because the label triggers in the minds of the learner the mental images associated to the previously learned concept. This results in establishing inappropriate links between the two concepts, which are from a mathematical perspective unrelated. Consequently the student is less successful in making use of the new definition to answer simple questions based on it.