WORD PROBLEMS AND ACTIVITIES –DESIGNING THE CURRICULUM

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The paper discusses alternative approaches to addition and subtraction word problems and argues that different approaches have different implications for mathematical literacy because of the differing emphasis on modelling reality. The role of modelling situations in terms of different semantic categories for real life problem solving is discussed. Analysis of textbooks in India shows absence of variety in the problems given with almost complete absence of the non-standard ones. Alternate trajectories for introducing children to the different categories of word problems are discussed in the context of the experimental practice going on Delhi

ALTERNATIVE APPROACHES

Activities in the teaching of mathematics have become accepted as an indispensable part of education in India, especially with the introduction of the Mathlab. Yet there is no consensus or even discourse about the possibly different role of the activities. Following Treffers, A. (1991) we can make a distinction between mechanical, structuralist, empiricist and realistic approaches. The dominant approach in India can be characterized as structuralist. While the mechanical and the structuralist approaches start from the structure of mathematics, the empiricist and realist start from the thinking process of the child.

Structuralism

In the structuralist approach the starting point emerges from the 'result of a logical analysis' of the finished structure of mathematics. This is what Freudenthal called as 'antididactical inversion' (Freudenthal, 1973, p103). Thus for example, if analysis shows that the algorithm for addition and subtraction involves carry-over and borrowing and it needs place-value, then instruction starts from place-value. In this structuralist approach, activities basically involve a mapping with materials that are isomorphous with the mathematical structure, such as in the standard use of Dienes blocks.

In the teaching of addition and subtraction, activities are generated from the vantage point of abstractly conceived 'addition' and 'subtraction', with number sentences such as 5 +3 = 8. To explicate this relationship sometimes contexts or materials are provided but the structure of the number sentence determines the structure of the activity. Word problems only come in for 'application' once proficiency has been achieved

Mathematisation

The alternate Realistic approach starts teaching of addition and subtraction from a situation meaningful to children. Learning involves learning to schematize the situation being explored. In other words, word problems are the introduction point for addition and subtraction. The introduction of a paradigmatic situation that needs to be *organized* for problem solving becomes the ground from which the meanings for the symbols get generated. Concrete materials and diagrams help to disembed the structures from the reality. This process of schematization that Treffers characterized as horizontal mathematisation, goes through many level raisings to finally reach the modern formalisms (Freudenthal, 1991, pp 41-42).

TYPES OF WORD PROBLEMS

Ability to solve word problems is closely related to mathematical literacy whose importance is being more recognised. (PISA, 2003, p 24). A review of what we know about word problems can help to evaluate the different methods of incorporating activities in the teaching of mathematics. Research has shown that the difficulty that children have to solve problems depends on the semantic structure and the nature of the unknown. (Carpenter, T.P., Hiebert, J. and Moser, J.M., 1983; Nesher, P., Greeno, J.G and Riley, M.S., 1982; Verschaffel, L. 1997).

Analysis of the word problems in five sets of textbook series from five publishers in India, including two government agencies, shows that the more difficult types are under-represented. (Table1). We followed the four way semantic categorization proposed by Carpenter et. al. (1983) They distinguished between the four categories of *Change*, *Combine*, *Compare* and *Equalize*. Although *Equalize* problems are included by many researchers in *Compare* problems, we retained the separate classification. “Equalize problems share characteristics of both *Change* and *Compare* problems. There is implied action on one of two given sets, but a comparison is also involved.” (Carpenter et. al., 1983, p 56). The *Change* word problems include those involving both Get-more and Get-less. Depending on the position of the Unknown, the Get-more semantic category could also involve either addition or subtraction.

Children’s responses to these word problems depend not only on the semantic categories but also on the position of the Unknown within the category, apart from other aspects related to number. Thus for example, *Change* problems involve some Starting quantity on which there is an action, which causes a Change and this produces a Resulting quantity. In the standard *Change* problem the *Start* and *Change* are given and the *Result* is unknown. Children are able to easily solve this standard *Change* problem. But in real life we often have to find out the *Change* that has occurred with knowledge of only the *Start* and the *Result* or to find the *Start*ing quantity with knowledge of only the *Result* and the *Change*. School children face difficulty with these types of problems. They also face difficulties with *Combine* problems in which the union set is given and the subset is unknown as compared to one where both the parts are given. (Nesher et. al. 1982, p 377). While *Compare* problems are considered more difficult than the *Change* and *Combine* problems, within *Compare* also, ones where the Difference is known with unknown Reference set or Compared set are more difficult. Those where the Reference set is unknown are the most difficult (Stern, 1993)

Textbook Series (Grades I to V)	Change Unknown Change Count (%)	Change Unknown Start Count (%)	Combine Unknown Part Count (%)	Compare Unknown Compare set Count (%)	Compare Unknown Reference set Count (%)	Total number of word problems
A	0 (0 %)	0 (0%)	12 (13.8%)	4 (4.6 %)	0 (0%)	87 (100 %)
C	0 (0 %)	0 (0%)	24 (15.2%)	5 (3.2%)	0 (0%)	158 (100%)
D	5 (3.5%)	0 (0%)	14 (9.8%)	1 (0.7%)	0 (0%)	143 (100%)
E	1 (1.3%)	0 (0%)	6 (7.9%)	1 (1.3%)	0 (0%)	76 (100%)
F	3 (1.2%)	0 (0%)	45 (17.4)	24 (9.3%)	1 (0.4%)	259 (100%)
Total						723

Table 1: Prevalence of difficult word problems in some textbooks in India

All the five textbook series together did not contain a single word sum in the semantic Category of *Change* with *Start Unknown* and there is only one word problem within the *Compare* category with *Unknown Reference* set. Even if the NCERT 2006 textbook series (incomplete) is included the situation does not change. The presence of the *Change* category with *Unknown Change* is also very meagre. It would appear that almost all the *Change* word problems are posed in the standard format. Across the Series and the Grades, about 50 % of the word problems belonged to the *Combine* category and about 35% to the *Change* category.

Word Problems and modelling

The low level of variation in the types of problems which are offered to children to solve can be taken as an indicator of the fact that they are not being prepared to model real life situations. Many of the textbooks including the recent NCERT textbook recommend use of keywords which are only surface characteristics, without exploring the possibilities for analogical reasoning to develop. (NCERT, 2006, p142) Yet studies have indicated that children are able, even without instruction, to model the semantic structure of the problem in however limited a manner. (Carpenter, 1986). It is this process of modelling to create an “intermediary” by which a complex reality is idealized to become accessible for formal mathematical treatment (Freudenthal, 1991, p 34), that needs to be considered in the teaching of word problems. It is in this context of the role of models, that we need to evaluate the different perspectives on the use of activities and word problems

CHALLENGES IN CURRICULUM DEVELOPMENT

If we *start* from the final goal of making the additive/subtractive structure of the word problems clear, then a particular pathway can be chosen. Following Fuson and other researchers (Fuson 1988, pp 247 -298), all the word problems, whatever their semantic structure can be understood in terms of the following schema:

Addend -Addend-Sum.

Depending on what is missing, the problem can be considered as addition or subtraction, independent of the semantic structure. Thus for example, a *Change-Get-Less* with the *Result* unknown would be a classic subtraction problem and a *Change-Get-More* with *Change* missing would be a less familiar subtraction problem. Yet both of them can be solved by counting up or indirect addition. This flexibility to choose the most efficient operation depending upon the number characteristic could be considered as a final goal.

With this goal in perspective, the activity to be conducted would be to teach the children to interpret different semantic types in terms of a part-part-whole schema.

If we start from the perspective of the familiar number sentences then the activities would have to be designed differently. With subtraction sentences such as, a - b = c, in which the subtraction sign is usually interpreted only in the context of 'take away', one option would be to develop activities so that the sign is also linked to *Compare* and *Equalize*.

Yet a different set of structuring of activities could emerge if we start from the perspective of children. It has been shown that even young children without instruction are able to model directly and solve not only the standard *Change* problems with unknown *Result*, but also where the *Change* is unknown. They do this by counting on with concrete materials, although normally it would be considered as a subtraction problem. Since for children the experiential reality of the different semantic categories is quite different we would have to facilitate the structuring of these realities. The limited evidence available shows that different types of models would facilitate dealing with the different types of semantic categories, rather than one common model (Fuson, 1988, p 287) and therefore one option would be to plan separate modelling activities.

Yet it is here that major methodological issues arise. Should one start from all the four (or three) types or should one start from two of them or from one typical case? In case we start from the typical case then when should the other semantic categories be introduced and in what manner? I believe that these are fundamental questions, the answers to which can radically modify our age-old practice of teaching addition and subtraction as time-honoured themes in primary education and perhaps replace them with problem solving in *Change*, *Combine* and *Compare* contexts. If we took *only* the perspective of the child and the context perhaps this would be the solution.

But if we consider that problem solving in the context needs to take into consideration the level raisings which has to be gone through to reach the final goal, then other aspects might emerge. It might be considered necessary to choose one typical semantic context and facilitate construction of the appropriate model that might further grow to cut its umbilical cord with the context to become a tool for thinking about other semantic contexts.

Experimental practice

In the curriculum development programme we have been doing in Delhi we have taken *Change* as the paradigmatic context introducing simultaneously both the Get-more and Get-less activities. There are many gray areas about the time and the manner in which the other type of contexts have to be introduced. We have worked broadly within a Realistic Mathematics Education (RME) framework adapting it as needed. In many cases we have taken the intermediate path and instead of the model emerging from the activity, we have incorporated the models as a part of the enactment of the situation with the symbols getting their significance through that incorporation.

Instead of the bus ride we took the story of Akshay to introduce the box and arrow symbolism of RME for depicting *Change* situation. A cubic box with 'chocolates' was kept on the table (but not opened). The story tells of Akshay who loves chocolates but was not allowed by the doctor to eat them due to his obesity. When he comes home from school one day, he sees the box on the fridge and telephones his mother at work to ask whether he could have one chocolate. She says no and tells Akshay that the chocolates are needed for giving two chocolates each to the two children who were expected in the evening for tuition. Later when Akshay calls back to say that there are more chocolates than what is needed and whether he could have one, the mother laughs and says that since he has been such a good boy he might have one. The story continues …. The story is narrated interactively and children join in to provide the numbers.

We have found that although this is the first introduction of the symbols, children are able to recount what happened at each of the six or seven steps of the story by following the 'inscriptions' on the board. What they like the most is the ending when at night watching television mummy and papa eat one each and "then there was.. zero chocolates left!" After a few sessions with variations of Akshay's story told by the children, the bus context and other *Change* contexts are introduced with the same notation. We could consider that with the "use of auxiliary devices, the transition to mediated activity radically reconstructs the whole mental operation" (Vygotsky, 1997, p 63). We also keep in mind the point made by Walkerdine that in the production of the chains of signification "action or objects do not make sense without a discourse in which to read them." (Walkerdine, 1988, p 123)

The paper would be also reporting some of the results from the interviews of children on their ability to deal with *Compare* word problems and reflect on the processes involved transforming the model from a model *of* the situation to a model *for* mathematical reasoning. (Gravemeijer, 1994, pp).

CONCLUSIONS

An analysis of the word problems used in some of the textbooks in India shows a very low presence of non-standard ones. The use of word problems in mathematics education needs to be considered from the perspective of mathematisation of the real world. This would require a major reorganisation of the manner in which activities with or without concrete materials are to be used in the classroom. The need for continued experimental practice to understand the trajectory for the development of situation specific models to tools for mathematical reasoning is suggested.

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