LEARNING LENGTH MEASUREMENT IN THE EARLY YEARS

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This paper draws on findings related to the learning of length from a major research project conducted in Victoria, Australia. Key elements of the project included the development of a one-on-one assessment interview and of growth points, or key stepping stones, in children's developing understandings. Results from a large number of interviews with children in the first three years of school indicate that children do progress through the conjectured growth points over time, that students' development of length concepts is quite independent of development of number concepts and that students need particular experiences associated with the learning of length, and that improvement in student understandings is teacher dependent.

OBJECTIVES AND SIGNIFICANCE OF THE STUDY

During elementary schooling, students learn about several measurement attributes including length, area, angle and volume. Although measurement concepts are complex and cause children a variety of difficulties, the basic idea of direct measurement appears simple (Wilson & Osborne, 1992). However, this apparent simplicity may lead teachers to underestimate the complex mental accomplishments involved (Stephan & Clements, 2003). This paper explores understandings within one measurement domain, Length, and considers implications for its teaching. We draw on findings from the *Early Numeracy Research Project* (ENRP), a three-year research project that was conducted in Victoria, Australia, involving teachers and children in the first three years of school in 35 project ("trial") schools and 35 control ("reference") schools (for details see Clarke, 2001). Two key components within the ENRP were

> the development of a research-based framework of "growth points" in young children's mathematical learning (in Number, Measurement and Geometry);
>
> a 40-minute, one-on-one interview, used by all teachers to assess aspects of the mathematical knowledge of all children at the beginning and end of the school year (February/March and November respectively);

The impetus for the ENRP was a desire to improve children's mathematics learning. To quantify such improvement a framework of pivotal *growth points* in mathematics learning, also described as primary stepping stones along the path to mathematical understanding, was developed. The one-to-one interview consisted of assessment tasks that were created to match the *growth points*.

UNDERLYING THEORETICAL FRAMEWORK

Much research on the learning of length is influenced by the work of Piaget and his colleagues. Piaget believed that children need to conserve length and have an understanding of transitivity to measure in an operational manner (Carpenter, 1976). Other key developmental concepts in coming to understand measurement include the idea of a unit, iteration, tiling, number assignment, partition,

comparison, additivity, and zero-point (e.g., Lehrer, Jaslow, & Curtis, 2003; Wilson & Osborne, 1992).

There have been a number of studies that confirm or challenge Piaget's findings (Carpenter, 1976; Kamii & Clarke, 1997; Lehrer, 2003; Stephan & Clements, 2003). Some of the challenges include the potential ambiguity of early measurement language, the possible possession by children of logical-mathematical structures before they can be demonstrated (Carpenter, 1976), and the difficulty of developing tasks to uncover internal logical-mathematical structures. As a result, the *Early Numeracy Research Project* based its framework on the more explicit aspects of measurement including comparisons, the idea of a unit, iteration, tiling, and number assignment.

The following describes the ENRP growth points identified for Length:

No apparent awareness of the attribute of length and its descriptive language. (*Not apparent*)

> This was used to categorise the responses of students who could not complete other items.

Awareness of the attribute of length and use of descriptive language (*Awareness of the attribute*)

> This refers to whether children show an awareness of the attribute (of length rather than colour, for example) and whether they use appropriate language. In order to understand the unit of measure children must understand the attribute being measured (Wilson & Rowland, 1993).

Compares, orders, & matches objects by length (*Comparing lengths*)

> Informed to some degree by the work of Piaget, measurement by direct comparison was then included in the next growth point. We considered that both the growth point and the task prompting direct comparison were inclusive and suggestive of conservation.

Uses uniform units appropriately, assigning number and unit to the measure (*Quantifying lengths*)

> This inferred use of non-standard but consistent units to quantify a length measurement. This includes the idea of iteration.

Uses standard units for estimating and measuring length, with accuracy (*Using standard units*)

> This refers to the use of formal units (e.g., cm) and an appropriate measurement tool (e.g., ruler) to quantify lengths.

Can solve a range of problems involving key concepts of length (*Applying*).

> This includes the iteration a 30cm ruler.

We did not include transitivity directly in the assessment framework, although it was included in the advice on teaching strategies.

The growth points were developed as a conjectured sequence. Recognising that students follow different pathways in their learning, the intention was to describe the learning trajectory of the majority of students.

The focus of interest in this presentation is on whether the data support the contention that the proposed length growth points describe goals for student learning over the first years of schooling, and on how teachers might help children reach these goals. The data reported here were collected in the year 2000, the second year of the project.

RESEARCH DESIGN AND PROCEDURES

Classroom teachers, trained in all aspects of interviewing and recording, conducted the one-on-one interviews, using a script and pre-prepared equipment. The data from this project arise from intensive interviews with large numbers of children, with trained interviewers, and experienced coders, with double data entry. The processes for assuring reliability of scoring and coding are outlined in Rowley and Horne (2000).

As far as possible in such situations, the profile of responses presented here can be taken as a reasonable representation of the way that students in Victorian schools would respond to such tasks.

In the interview there were four questions that addressed length, using a range of equipment including string, paper clips and a ruler. The interviewers proceeded through the interview in order, but moved directly to the next domain, that is, Mass, if the student answered a question incorrectly. A coding rubric was used to score the students' responses.

FINDINGS

The following is a summary of the key results from this aspect of the research. The data substantiating these results will be included as part of the presentation.

One issue of interest is whether each of the conjectured growth points is needed. Data suggested that the growth point *Awareness of the Attribute* is not needed. The two key growth points at the School entry level are *Comparing length* and *Quantifying length*, and data indicate that these are clearly needed to describe both learning and growth potential for these students.

The issue of the use of non standard units has been the subject of some debate (e.g., Stephan & Clements, 2003; Wilson & Rowland, 1993). The ENRP results suggest that it is indeed important to use non standard units (e.g., handspans, matches, unifix). Using them for teaching in a way that bridges the counting of units and the use of standard units allows the teacher and children to focus on student reasoning rather than purely counting (see also Lehrer et al., 2003; McClain, Cobb, Gravemeijer, & Estes, 1999), thus helping children to more meaningfully use tools such as rulers

We were interested also in whether the growth points represent a sequence. It is not possible to infer a sequence from individual scores, but comparisons from one assessment to the other indicate that the students progress through these points, over time, in this order.

A further issue that arises is the relationship between domains. For example, by comparing data from the domain of Length, with that from the domain of Counting, it becomes apparent that there is little direct connection between children's success in counting 20 objects and their success on measuring an object with four non-standard units. Indeed, the successful use of non-standard units requires a variety of understandings including unit iteration (to think of the paperclip as part of the length of the straw and to place the paperclip repeatedly along its length), and tiling (to understand that units must fill the space with no gaps or overlaps) (e.g., Lehrer, 2003; Stephan & Clements, 2003), as well as counting and correct identification of the unit. The ENRP data suggest that the development of length concepts is quite independent of the development of number concepts, and that a balanced program needs length included. To develop to the next growth point students need

particular experiences associated with the learning of length, rather than general mathematical development.

Interview data indicated that some teachers were more successful than others in their students achieving growth for length. For example, in a study of results from children in their first year of school, it became apparent that there was great variation in the number of children who progressed in their Length growth points from March to November. These were very noticeable differences and were unlikely to have occurred due to chance factors. An examination of the schools and other factors indicated that neither being effective nor being less effective teachers in terms of promoting improvement was dependent on school size, socio-economic community, student language background, or years of experience of the teacher. This suggested it is possible for students at this level to move through the growth points but it is teacher dependent. In the presentation we will share some insights gained about the characteristics of these effective teachers.

Among other things, the Early *Numeracy Research Project* aimed for the growth points and interview to emphasise important concepts and skills in early mathematics in a form and language readily understood by and capable of being retained by teachers, and to form the basis of planning and teaching. Teacher responses indicate that the project was successful in meeting these goals. The growth points and interview were reported to have allowed teachers to see more clearly where a child was in each domain, to be more aware of the various needs of children, and therefore to inform planning and teaching.

Additional Information

The Early Numeracy Research Project (ENRP) was a collaborative venture between Australian Catholic University, Monash University, the Victorian Department of Education and Training, the Catholic Education Office (Melbourne), and the Association of Independent Schools Victoria. The project was funded in 35 project ("trial") schools and 35 control ("reference") schools. The views presented here are those of the authors.

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