PROPORTIONAL REASONING OF ADOLESCENT AND ADULT HIGH SCHOOL STUDENTS

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This paper reports part of an ongoing investigation on the strategies used by subjects of different ages and schoolings when faced to different kinds of ratio-comparison tasks. We are here concerned with the comparison of the ability for proportional reasoning between two groups of the same schooling and different ages: adolescents and adults who are high school students. Normally, at least in developed countries, most adults have more schooling than young subjects, and therefore the differences among adolescents and adults cannot be ascribed to age or schooling alone. However, in this study the fact that all of the subjects are students at the same educational level permits to associate the differences with the age of the subjects, and not with their schooling.

TYPES OF PROPORTIONALITY PROBLEMS

In the complex setting of proportional reasoning research, several ways have been put forward to classify the problems that can be proposed to subjects. These ways may be in turn grouped in classifications according to at least three issues that affect the subjects' responses: 1) the *task*, 2) the *context*, and 3) the *numerical structure*.

The *task* that subjects have to accomplish was classified by Tourniaire and Pulos (1985) as "missing value problems" or "ratio comparison problems". To this basic classification other researchers, such as Lesh, Post and Behr (1988), later added more categories. In the research reported in this paper only ratio comparison problems are considered.

Among the classifications according to the *context*, Freudenthal (1983) distinguished couples of a) expositions, b) compositions, and c) ∑-constructs; Tourniaire and Pulos (1985) set apart d) physical, e) rate, f) mixture, and g) probability problems; and other authors, among which Lamon (1993), have distinguished h) well chunked measures, i) part-part-whole problems, j) associated sets, and k) stretchers and shrinkers. Although each of these classifications and categories corresponds to particular views and goals, categories a), d), e), h), and j) can all be considered as one and the same because they all involve two different quantities; the difference between d) and e) lies in the fact that the latter are word problems, and the difference between h) and j) lies in how familiar the subject finds them. Categories b) and i) can be considered as one, because they involve one quantity; f) and g) are in the same case and the difference among them may be considered important. Finally, categories c) and k), which are problems of a geometrical nature, can be considered as one. The left column of Table 1 displays the condensed classification of context resulting from these considerations. It may be noted that in rate problems two quantities are at stake, and thus there is an intensive quantity formed (Schwartz, 1988).

As examples of the first three types, see Figures 1, 2 and 3. This research does not deal with geometrical problems.

Rate problems: couples of expositions		Two quantities (and/or an intensive quantity surging from them)
Part-part-whole problems: couples of compositions	Mixture	One quantity
	Probability	One quantity
Geometrical problems: couples of ∑-constructs		Two quantities

Table 1: Problem classification according to context

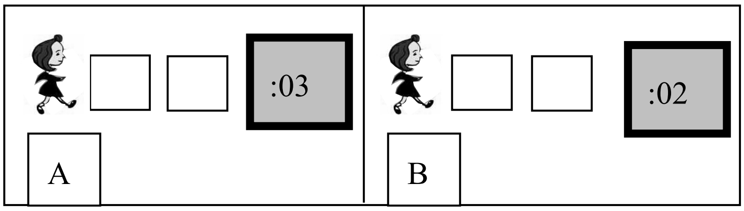

Figure 1. Example of a Rate problem: Which of the girls walks faster? (Two quantities: the amount of blocks, represented by squares, and the time taken to walk them, represented by the numbers in the box)

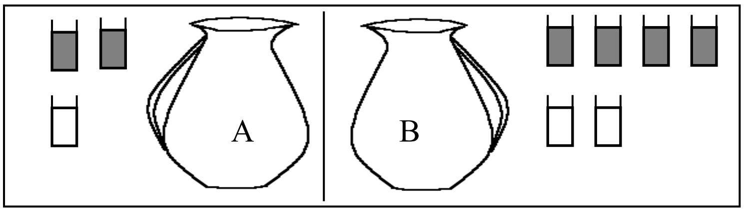

Figure 2. Example of a Mixture problem: In which jar does the mixture of concentrate and water have a stronger taste? (One quantity: the amount of glasses. Problem taken from Noelting, 1980)

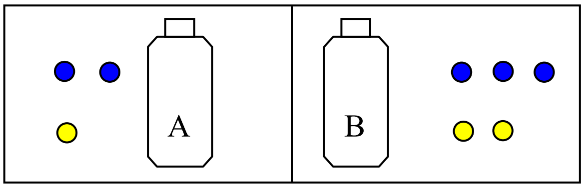

Figure 3. Example of a Probability problem: If bottles are shaken with marbles inside, in which bottle is a blue marble more likely to come out at the first try? (One quantity: the amount of marbles)

The third issue is the *numerical structure*. In a ratio or rate comparison there is always a foursome: four numbers stemming from two "objects" (A and B), in each of which there is an antecedent (e.g. blocks, concentrate glasses, blue marbles) and a consequent (e.g. minutes, water glasses, yellow marbles). Alatorre (2002) proposed a classification of all possible foursomes in 86 different situations that can be grouped in three difficulty levels, labelled L1, L2, and L3, which depend on the kinds of strategies that can be used to correctly solve the comparison:

- Level L1 consists of all the numerical situations where, in addition to proportionality strategies **(PS)**, other correct strategies may be used. An example of L1 is the foursome of Figure 1; it can be solved saying that girl B walks faster because she takes 1 minute per block, whereas girl A takes 1½ minute per block (PS), or saying that girl B walks faster because she walks the same blocks than girl A but in less time.

- Level L2 consists of all the numerical situations of proportionality that need a PS to be correctly solved. An example of L2 is the foursome of Figure 2; it can only be solved by a PS, for instance saying that both jars taste the same because they both have twice as many concentrate glasses than water glasses.
- Level L3 consists of all the numerical situations of non-proportionality that need a PS to be correctly solved. An example of L3 is the foursome of Figure 3; it can only be solved by a PS, for instance saying that in side A it is more likely to get a blue marble because it has two blue marbles for each yellow one, whereas side B lacks a blue marble to have the same relationship.

METHODOLOGY

A case study was conducted in a suburban lower class area of Mexico City with 10 subjects: five adolescents aged 13 to 15 who attended the 8th or 9th grade in a high school, and five adults aged 29 to 37 who attended the 8th or 9th grade in an open system for adults. The individual videotaped interviews lasted between 60 and 90 minutes.

Eight problems were designed (4 Rate, 2 Mixture, and 2 Probability problems), and 15 questions that varied according to numerical structure (five in each of the difficulty levels L1, L2, and L3). During the interviews, subjects were posed all of the problems in some of the 15 questions, covering at least 2 of each level. Each time, the subjects were asked to make a decision (side A, side B, or “it is the same”) and to justify it.

927 answers were obtained, which were categorized according to the strategy or strategies that subjects used in them (a complex system for this categorization was used; see Alatorre, 2002). The strategies were then classified according to correctness. As it was said before, PS are always correct but they are not the only correct strategies that may be used; in L1 other more simple strategies can also be correct; on the other hand, not all attempts at using a PS are necessarily correct, because mistakes can occur in the application of a PS.

After this a quantitative analysis was performed. For this, the percentage of correct answers was considered in different groups of answers (e.g., for each subject and difficulty level).

With such quantification, a developmental model was proposed by Alatorre and Figueras (2005), based on experimental data. According to it, L1 situations are first dealt with, generally using non-PS strategies, and at that first stage L2 and L3 are numerical situations that are difficult because the subject has trouble applying PS. PS are first learnt in proportionality situations (L2), and thus in a second stage or moment subjects can deal with L1 problems and start being able to solve the L2 problems. Only when a certain expertise is attained in these numerical situations (third moment) can PS start to be applied in non-proportionality situations (L3). Finally (fourth moment), subjects can deal with all three difficulty levels. This evolution is depicted in Figure 4.

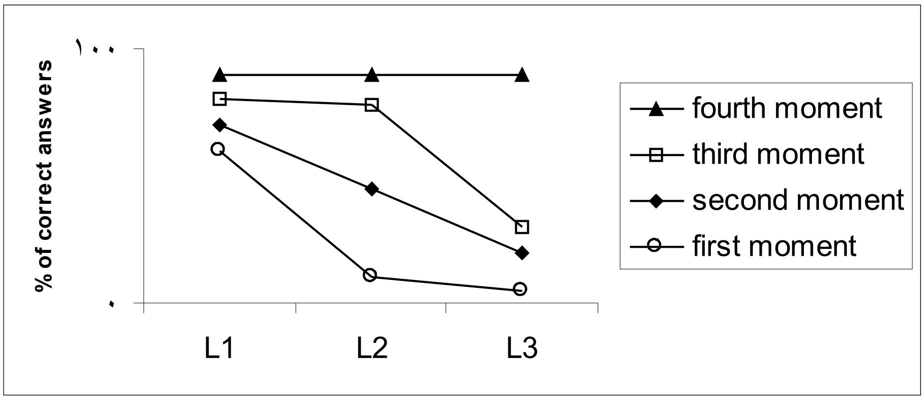

Figure 4. Developmental model

However, the model applies only within a certain type of context. It is frequent to see for instance subjects who at a given time are in the fourth moment in Rate problems, at the third moment in Mixture problems, and at the first moment in Probability problems.

ANALYSIS OF RESULTS

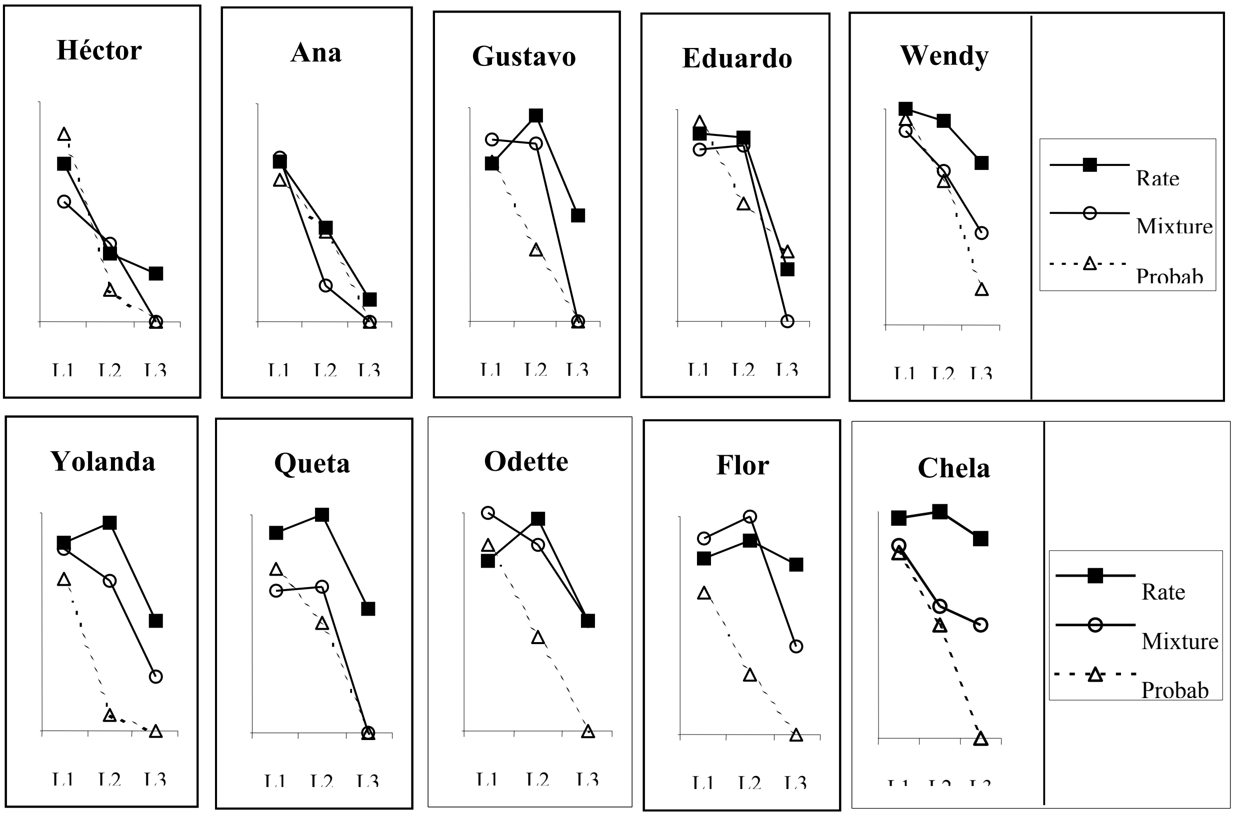

Figure 5. Percentage of correct answers in different contexts and difficulty levels: results of the five young students (upper row) and five adults (lower row)

A graph like the four lines of Figure 4 was obtained with the data of each subject's answers to the questions of different context types and different difficulty levels. Each subject's graph thus obtained (Figure 5) describes their ability to solve the different ratio-comparison problems at the moment of the interview.

It can be seen that there is a wide variety of responses. The lowest results correspond to two young subjects (Héctor and Ana); it is noticeable that none of the five adults showed results as low as them. Generally speaking, subjects obtain their best results in L1 and the worst ones in L3; also, Rate problems are the easiest and Probability ones are the most difficult. The easiest case is Rate problems in L2, and the most difficult case by far is Probability problems in L3 (see Figure 3).

Along with this quantitative analysis, a qualitative analysis may be performed by glancing at the strategies used. At the difficulty level L1, where different sorts of correct strategies may be applied, adult subjects used more PS than young students, and the latter used more non-PS strategies. The adult subjects produced most of the overall obtained PS's in L2 and L3 (and thus the correct answers, since at those two levels the only correct strategies are PS). At L3 in Mixture problems, all of the adults except one (Queta) could use PS, but only one of the adolescents could (Wendy; see also Figure 5). On the other hand, also at L3 but in Probability problems, none of the adults could produce PS whereas two of the adolescents could, at least in some instances (Eduardo and Wendy). As for incorrect strategies, not shown in Figure 5, the most outstanding results are the following:

- In L2 and L3 young subjects used incorrect additive strategies more often than the adult subjects.
- In L2 all of the adults' attempts at PS were successful but not all of the young subjects'. This means that young subjects often made mistakes while trying to use a PS; for instance, they calculated the quotient antecedent/consequent on one side but the quotient consequent/antecedent on the other one. This did not happen to adult subjects.
- In L3 adult subjects made more incorrect attempts at PS than young ones. Adults did make such mistakes as previously exemplified in L3, that is in situations of non-proportionality, which are, as Figure 4 shows, the most difficult, but young subjects made fewer mistakes. This is because many young subjects did not even try to use a PS in L3.

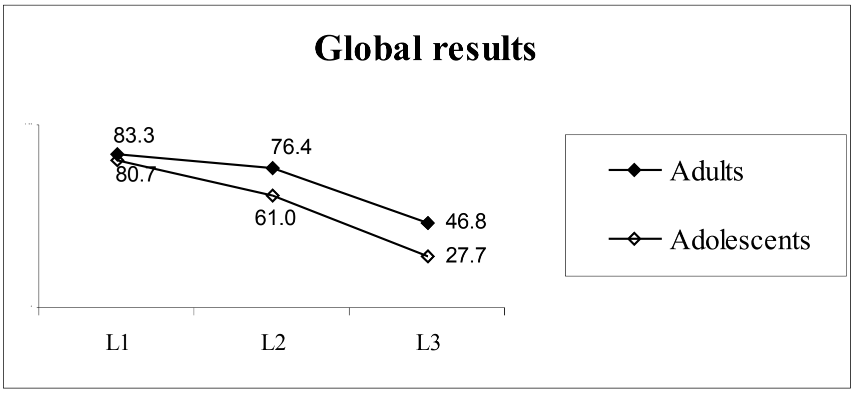

Figure 6. Comparison of both groups of subjects in levels L1, L2, L3
(percentages of correct answers in each difficulty level)

In short, the five adult high-school students interviewed obtained globally better results than the young ones. Although at level L1 the difference is practically null, at levels L2 and L3 it reaches respectively 15.4% and 19.0% (see Figure 6). This may be due to the fact that it is easier for adult subjects to use proportional reasoning when no other strategy can solve the problem (L2 and L3); young ones often err in the application of PS and also make other mistakes.

CONCLUSIONS

The obtained results can throw some light into the differences between adolescents and adults in proportional reasoning. Proportional reasoning is *taught* (although, as we have seen, not necessarily *learnt*) at school, and therefore people with more schooling tend to obtain better results at problems calling for proportional reasoning than less schooled people; this is usually the case while comparing adults with young people. In this study, however, the fact that all ten subjects have the same schooling permits to associate the observed differences with the age of the subjects rather than with their schooling.

Although some differences are small, these five adults obtained generally better results than adolescents. They seem to have learnt how to use proportional strategies in proportionality (L2) as well as in non-proportionality (L3) situations, with the possible exception of probability problems, where adolescent students obtained better results than the adult ones. It can be conjectured that except for probability and at least in comparison with the young subjects, adult students have learnt to use proportional reasoning through life experience: Proportional reasoning is not only learnt at school, but also through daily life.

Although the ten subjects who participated in this study do not form a representative sample of adolescent and adult high school students, and therefore no inference is strictly valid, the observed differences in this case study allow us to hypothesize that the daily life experience should be taken into account in the teaching of proportional reasoning, especially with adult students.

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